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# **Supplementary Guidance Note 3A - Probabilistic contingency estimation**

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Email: [costestimation@infrastructure.gov.au](mailto:costestimation@infrastructure.gov.au)

Website:

[https://investment.infrastructure.gov.au/about/funding\\_and\\_finance/cost\\_estimation\\_guidance.aspx](https://investment.infrastructure.gov.au/about/funding_and_finance/cost_estimation_guidance.aspx)

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# 1. Introduction

## 1.1. Context

The Department of Infrastructure, Transport, Regional Development, Communications and the Arts (the department) provides and maintains cost estimation guidance which is intended to inform and assist proponents in improving and establishing cost estimation practices. The suite comprises the following volumes:

- Guidance Note – Overview
- Guidance Note 1 — Project Scope
- Guidance Note 2 — Base Cost Estimation
- **Guidance Note 3A – Probabilistic Contingency Estimation**
- Supplementary Guidance Note to 3A
- Guidance Note 3B – Deterministic Contingency Estimation
- Guidance Note 4 – Escalation

Under the policy settings a probabilistic cost estimation process must be used for all projects, for which Commonwealth funding is sought, with a total anticipated outturn cost (including contingency) exceeding \$25 million.

## 1.2. Supplementary guidance to 3A

**Guidance Note 3A** outlines the department's expectations and application of probabilistic techniques for projects over \$25 million. Due to the nature and complexity of the topic, **Guidance Note 3A** provides only a brief description of the fundamental techniques and key information required to undertake probabilistic modelling. This supplementary guide provides further information as well as worked examples of each of the techniques recommended in **Guidance Note 3A – Probabilistic Contingency Estimation**.

Additional useful guidance on cost estimation practices, to the extent that they do not contradict the department's guidance, may be found in individual agency cost estimation guidance or manuals, and in the guidance provided by professional associations such as Engineers Australia Risk Engineering Society (RES), AACE International, Project Management Institute, or in risk analysis textbooks.

## 1.3. Objective and scope

The objective of this guidance note is to provide more detailed explanation and discussion of the technical content of **Guidance Note 3A – Probabilistic Contingency Estimation**. It is tailored to practitioners developing estimates for projects for which Commonwealth funding is being sought under the Infrastructure Investment Program (IIP), generally for road and rail infrastructure projects, but the principles are applicable to all project types.

It covers the following topics:

- **Introduction and background** – discussion on the background and reasons for undertaking a quantitative risk analysis on infrastructure projects.
- **Quantitative Risk Analysis (QRA) Workshops** – discussion on the techniques and procedures for undertaking QRA workshops and for eliciting expert opinion, as part of preparing a probabilistic cost estimate.

- **Cost Risk Model structure** – outlines the principles to assist analysts to build efficient and realistic risk models and is accompanied by worked examples.
- **Cost risk models** – discussion and examples of several quantitative cost risk analysis approaches.
- **Interpretation of output results** – discussion on how to analyse, understand and communicate the output of a probabilistic risk analysis.

This document contains appendices providing further detail on:

- Derivation of risk factors
- Simulation and statistical analysis
- Correlation
- Choice of probability distribution function
- Aggregation of inputs and number of line items to model
- Number of iterations to run in a simulation
- Common Monte Carlo simulation errors
- Alternate approaches to eliciting expert opinion

The appendices, while essentially stand-alone documents, should be read in the context of the guidance note suite.

It should be noted that probabilistic cost estimating is best suited to individuals with certain skills and training. Many organisations use facilitators to work with their estimators and engineers to assess contingency requirements as, in addition to some specialist modelling capabilities, an understanding is required of common biases and how to work with a team to arrive at a realistic view of cost uncertainty. This document is not intended to enable estimators with no prior experience to perform probabilistic estimating, there is more to the process than being able to build and run a probabilistic model.

Ideally, those developing probabilistic estimates for projects seeking Commonwealth funding should have suitable qualifications and experience such as:

- Tertiary qualifications in decision science, probability and statistics, engineering or similar.
- AACE International Certified Cost Professional (CCP), Decision & Risk Management professional (DRMP), or similar.

## 2. Background

### 2.1. The probabilistic nature of estimates

By their very nature, estimates are uncertain projections of future events and circumstances. Cost and schedule estimating is an integral part of the project management process as organisations use these estimates for planning purposes such as options appraisal, CBA, and resource and budget allocation.

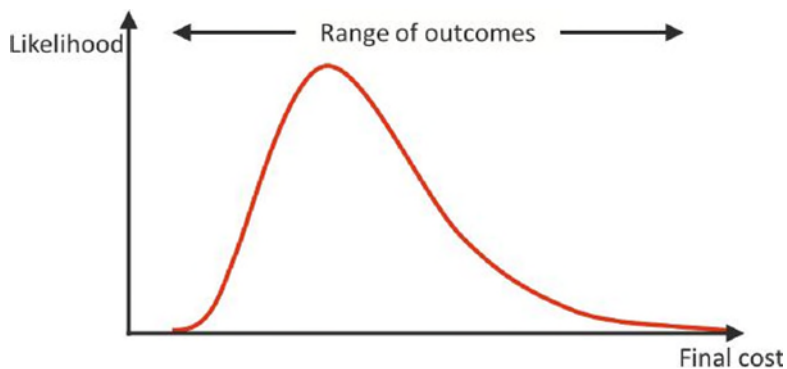
The word “estimate” itself implies uncertainty, so an estimate is not completely specified by a single number. A distribution of possible outcomes is required to provide a level of confidence in an estimate. The distribution of possible cost outcomes for a project is represented by an estimate’s probability distribution that is calculated, or simulated, through the application of probability and statistics. These are crucial



concepts as the laws of probability are the most powerful tools of risk management that we have at our disposal<sup>1</sup>. The laws of probability will allow us to understand or make sense of future outcomes.

Since estimates cannot guarantee a specific outcome with complete certainty, distributions of possible cost outcomes are represented by an estimate's probability distribution, which is calculated or simulated through the application of probability and statistics as per **Figure 1**.

**Figure 1: Typical cost probability distribution**



## Uncertainty and risk

It can be considered that there are two universal axioms in relation to estimating<sup>2</sup>:

You can't accurately estimate anything if you don't understand what it is.

All estimates are uncertain. The estimator/analyst's job is to minimise the uncertainty, as far as that can be done at each stage in a project.

If something is only partially understood, any estimate made about it will be uncertain, early estimates can be wrong by very large percentages which is related to the limited information available at the time the estimate was made. The objective is not to produce an estimate that is highly representative of the future completed project on day one (which is all but impossible), but to produce a sequence of estimates that become closer and closer to the actual outturn cost as the project proceeds. The accuracy of the estimates at a given point in a project's development will reflect what is known about the project at that stage.

Estimates should be made independently of budgets or committed funding, trying to match cost estimates to budgets could cause cost overruns. However, it costs money to wait, to study a proposed project and to collect information and may not be possible due to time or other constraints, often an estimate must be made and converted to budgets while information is still unknown. At the point a commitment is to be made, a realistic quantification of the risks must be undertaken and sufficient contingency and risk allowances included on top of the base estimate to ensure that the level of funding and other commitments made, reflect the organisation's risk appetite.

## 2.2. Types of risk

ISO31000 defines risk as "the effect of uncertainty on objectives". However, many alternate definitions are available within the literature. The majority emphasise that risk is concerned with the chance of undesired events, usually within a specific time frame, and almost universally identify probability as the key metric in this context.

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<sup>1</sup> Bernstein, P., (1998) *Against the Gods: The Remarkable Story of Risk*, John Wiley & Sons Inc, New York

<sup>2</sup> Adapted from Stump E, (n.d.) *Breakings Murphy's Law: Project Risk Management* (downloadable at [galorath.com/wp-content/uploads/2014/08/stump\\_breaking\\_murphys\\_law.pdf](http://galorath.com/wp-content/uploads/2014/08/stump_breaking_murphys_law.pdf))

Defining risk mathematically also varies across disciplines. The value of a particular risk, is not necessarily simply calculated as the product of probability and consequence<sup>3</sup>.

In the context of risk analysis and identification, uncertainty can be thought of in two ways. The first is a sense that the quantity we are trying to estimate (such as the volume or cost of earthworks) has some uncertainty attached to it. The second is risk events – random events that may or may not occur which are of interest to us.

Terminology to distinguish between these two types of risk varies. A common distinction is to use the terms inherent risk (to distinguish between uncertainty that is certain to have some effect) and contingent risk (uncertainty that might have no effect or could have an effect - something like an event). As another example, the US Federal Highway Administration Guide to Risk Assessment and Allocation for Highway Construction Management<sup>4</sup> notes that some risk may be measured incrementally and continuously, whereas other risks are discrete

AACE International typifies risk as falling into one of two categories<sup>5</sup>, systemic risks, and project-specific risks. Systemic risks are seen as being associated with the culture, capabilities and practices of the engineering, estimating and construction communities that deliver projects so they affect all projects in a particular sector, while project specific risks depend on the details of an individual project.

## Systemic risks

The term systemic implies that the risk is an artefact of the system that conceives of, estimates and plans, and then implements the project. It includes features such as the quality of the management team and forecasting methods and the commercial arrangements under which the work will take place. Research suggests that the impacts of some of these risks are measurable and predictable for projects developed within the same system. They are generally known even at the earliest stages of project definition where the impact of these risks tend to be highly dominant<sup>6</sup>. The challenge is that the link between systemic risks and cost impacts is complicated as well as being stochastic in nature, which means that is very difficult to understand and to directly estimate the aggregate impact of these risks at an individual line-item level.

In general, systemic risks are owner risks, the owner is responsible for early definition, planning and so on, which are risks that cannot be readily transferred to contractors. Typical systemic risks for infrastructure projects include:

- Project definition:
  - Geotechnical requirements
  - Engineering and design
  - Safety and environmental risks and/or approvals
  - Planning and schedule development
- Project management and estimating process:
  - Estimate inclusiveness
  - Team experience and competency
  - Cost information availability

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<sup>3</sup> Hayes K (2011) Uncertainty and Uncertainty Analysis Methods (downloadable at <https://publications.csiro.au/rpr/pub?pid=csiro:EP102467>)

<sup>4</sup> Federal Highway Administration (2006) Report FHWA-PL-06-032 Guide to Risk Assessment and Allocation for Highway Construction Management

<sup>5</sup> AACE International (2008) RP No. 42R-08 Risk Analysis and Contingency Determination Using Parametric Estimating

<sup>6</sup> Hollmann, J (2016) Project Risk Quantification: A Practitioner's Guide to Realistic Cost and Schedule Risk Management, Probabilistic Publishing, Gainesville, FL

- Estimate bias

## Project-specific risks

Project-specific risks are specific to a particular project and therefore the impacts of these risks may not be the same from one project to the next. Measures of these risks will generally not be known at the earliest stages of project definition. The link between project-specific risks and cost impacts is more easily understood than systemic risk and can make it easier to estimate the impact of these risks on specific items or activities. These risks are amenable to individual understanding and quantification using expected value or simulation techniques. For example, it may be possible to estimate the impact of wet weather on earthworks activities reasonably accurately or at least appreciate the range of impacts it could have. Typical project specific risks may include:

- Weather
- Site subsurface conditions
- Delivery delays
- Constructability
- Resource availability including materials, plant and labour.
- Project team issues
- Environmental or heritage issues
- Legislative issues
- Quality issues

For the purposes of this guidance note, the AACE typologies as just described are preferred and will be utilised when distinguishing between risk types.

## 2.3. Quantitative risk analysis

Quantitative risk analysis and modelling provides a means of<sup>7</sup>:

- Describing the detailed mechanisms at work in the way uncertainty affects a project.
- Evaluating the overall uncertainty in the project and the overall risk placed on stakeholders.
- Establishing targets, commitments, and contingency amounts consistent with the uncertainty the project faces and the risk that managers are willing to accept.
- Exploring the relationship between detailed instances of uncertainty and an overall level of risk, to inform risk management resource allocation and other measures that may be taken to optimise the project.

There are several ways to quantify risks or to estimate contingency. Each method has strengths and weaknesses. Some have been shown to lead people into poor practices. AACE International Recommended Practice No. 40R-08 “Contingency Estimating – General Principles”<sup>8</sup> notes that any methodology developed or selected for quantifying risk impact should address the following general principles:

- Meet client objectives, expectations and requirements.
- Forms part of and facilitates an effective decision or risk management process.
- Fit-for-use.

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<sup>7</sup> Cooper D, Bosnich P, Grey S, Purdy G, Raymond G, Walker P, Wood M, (2014) Project Risk Management Guidelines: Managing Risk with ISO 31000 and IEC 62198 2nd Edition, John Wiley and Sons Ltd, Chichester

<sup>8</sup> AACE International (2008) RP No. 40R-08 Contingency Estimating – General Principles, AACE International, Morgantown, WV

- Starts with identifying the risk drivers with input from all appropriate parties.
- Methods clearly link risk drivers and cost/schedule outcomes.
- Avoids iatrogenic (self-inflicted) risks.
- Employs empiricism (experience of past projects).
- Employs experience/competency.
- Provides probabilistic estimating results in a way that supports effective decision making and risk management.

Quantitative risk analysis is the process of identifying and analysing critical project risks within a defined set of cost, schedule, and technical objectives and constraints. It assists decision makers to balance the consequences of failing to achieve a particular outcome against the probability of failing to achieve that outcome. Its purpose is to capture uncertainty in such areas as cost estimating methodology, completeness and reliability of the information available, technical risk, and programmatic factors in order to go from a deterministic point estimate to a probabilistic estimate.

A credible base estimate (**see Guidance Note 2 – Base cost estimation**) is the key starting point in generating a risk-adjusted estimate and the development of confidence intervals. Risk analysis provides an analytical basis for establishing defensible cost estimates that quantitatively account for project risks. It is important that this analysis be continuously reviewed and updated as more data become available during the development of designs, estimates and plans.

Throughout this process, interactions take place between the following actors<sup>9</sup>:

- The stakeholders (individuals or organisations that are affected by the outcome of a decision but are outside the organisation doing the work or making the decision).
- The risk analysts (individuals or organisations that apply probabilistic methods to the quantification of risks and performances).
- The subject matter experts (individuals or organisations with expertise in one or more topics within the decision domain of interest).
- The technical authorities
- The decision-maker

Given the presence of uncertainty, we know that the actual outcome of a particular decision alternative will fall within a range but we do not know exactly where. Risk analysts must therefore model all possible outcomes of interest, accounting for their probabilities of occurrence, in terms of the scenarios that could be experienced. This produces a probability distribution of outcomes for each alternative (**Figure 1**).

If the uncertainty in one or more performance measures (for example, cost, schedule, safety and/or travel time benefits, etc.) prevents the decision-maker from assessing important differences between alternatives, then more information may need to be gathered and the analysis iterated in order to reduce uncertainty. The iterative analysis process stops when the level of uncertainty does not preclude a robust decision from being taken.

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<sup>9</sup> Zio & Pedroni (2012) Overview of Risk-Informed Decision-Making Processes (downloadable at <https://www.foncsi.org/fr/publications/collections/cahiers-securite-industrielle/overview-of-risk-informed-decision-making-processes/CSI-RIDM.pdf>)

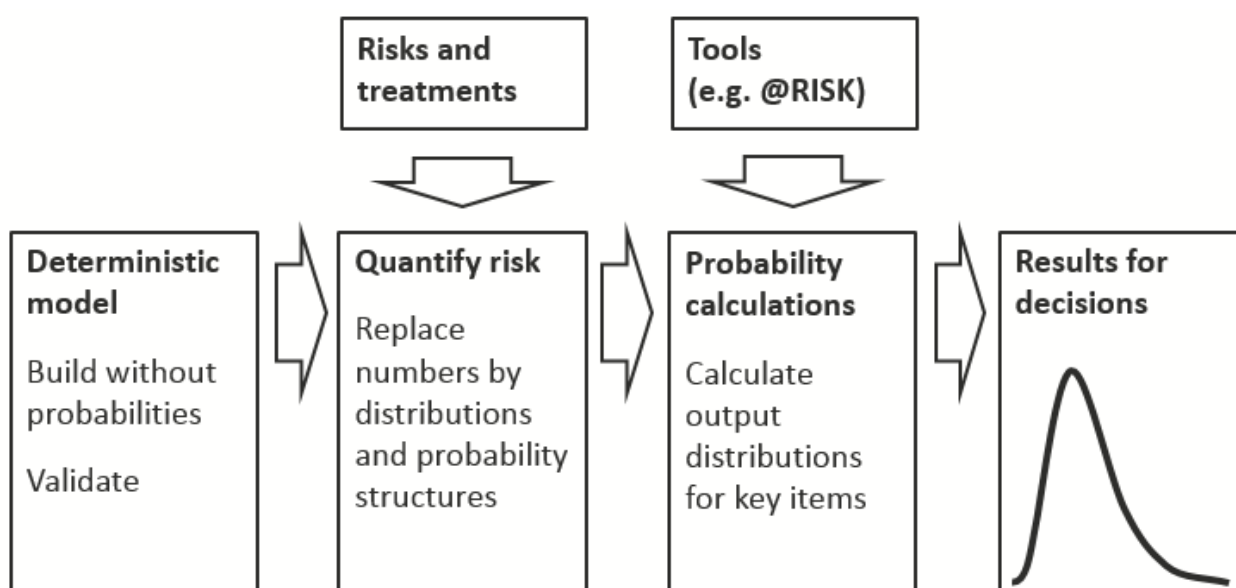
## Robust decision

A robust decision is based on sufficient technical evidence and characterisation of uncertainties to determine that the selected alternative best reflects decision-makers' preferences and values given the state of knowledge at the time of the decision, and is considered insensitive to credible modelling perturbations and realistically foreseeable new information<sup>10</sup>.

The final output of the quantitative risk analysis process is a project cost Cumulative Distribution Function (CDF, or "S" curve) that estimates the probability of exceeding a given cost. This enables decision-makers to understand the level of risk associated with each potential cost outcome and set an appropriate budget with the level of risk the organisation is prepared to accept.

**Figure 2** shows a simplified outline of the quantitative risk analysis process, noting that it will almost always be iterative in nature.

**Figure 2: Outline of the quantitative risk analysis approach – redrawn from Cooper et al (2014) – copyright**



Project costs are generally subject to more than one source of risk so a risk analysis must be able to model the combined effect of several risks. The analysis of risk, using computer software, is usually undertaken through "Monte Carlo" or simulation analysis.

The Monte Carlo simulations uses repeated random sampling to obtain numerical results and is useful in assess systems with multiple interacting risks. The technique is generally used to solve problems which are too complex or too cumbersome to solve mathematically. Other approaches may be used to model project cost risk, provided they estimate contingency, are mathematically sound, and are consistent with best risk management practice.

Monte Carlo simulation is simply a way to evaluate a model with uncertain parameters. There are several approaches to preparing a model of project cost risk that can be evaluated this way. While there is no one 'right' way to quantify risks or estimate contingency, any method must be mathematically sound, and be consistent with best risk management practice. Some methods do not satisfy either of those two criteria.

There are two major practical considerations regarding the extent to which Monte Carlo simulation can be used to estimate project costs. The first is the issue of data availability. Data availability is likely to vary both

<sup>10</sup> NASA (2010) Risk-informed Decision Making Handbook (NASA/SP-2010-576). Technical report, NASA

between individual projects, and across the different sources of risk within any one project. This limitation is discussed in detail at Section 5 and can be overcome through appropriate techniques to elicit expert opinion which avoid bias and the other pitfalls that can interfere with obtaining realistic assessment of uncertainty. The second consideration is the extent of correlation between those variables selected for risk analysis. Projects are rarely subject to only one source of risk, which is why more than one variable at a time is modelled in the Monte Carlo simulation exercise<sup>11</sup>, and statistical complexities can arise in the relationships between variables. Where variables may be thought to be related, the extent of correlation between them needs to be taken into account.

The cardinal rule of Monte Carlo simulation can be expressed as: “Every iteration of a risk analysis model must be a scenario that could physically occur”<sup>12</sup>. The model must therefore be prevented from producing, in any iteration, any combination of values that could not possibly materialise. One way in which this can occur is by not accounting for the interdependency between the model inputs.

It is often poorly understood that valid Monte Carlo simulation requires that the dependencies between model inputs be defined. In fact, being able to define the relationships between uncertain input distributions was arguably the key motivating factor behind the development/invention of what is now known as the Monte Carlo method by Stanislaw Ulam in the late 1940’s<sup>13</sup>.

As an example, if two variables in an analysis are market rates for locally produced concrete and market rates for locally sourced construction plant, it will not generally be realistic to select a value for one that is at the high end of its possible range and a value for the other at the low end of its possible range. Similar pressures of supply and demand in the local economy will tend to drive them both towards the high end, the low end or the middle of their ranges rather than one being high and the other low. Misapplication of Monte Carlo simulation, or poor modelling technique, is likely to see such implausible situations arise.

The correlation between variables in any analysis must be understood, however as explained at Section 4 and at Appendix C this can be difficult to achieve in practice. As well as the difficulty of addressing correlation, and in part because of not accounting for it, some approaches in common use, such as line-item ranging, if not applied correctly will produce an unrealistically narrow range of possible costs. They will understate the required contingency at higher confidence levels, as well as understate the probability of achieving lower cost outcomes. Many of the inherent difficulties within line-item ranging approaches are well recognised but attempts to overcome them, by building in complex overlapping correlations, usually result in models that are far more complicated than they need be without necessarily making them any more realistic.

## 2.4. The aggregation problem

The choice of level to which detailed costs are aggregated into components of an analysis and the evaluation of correlation between the components are critical to the validity of results of a simulation. Level of aggregation refers to the level of detail with which an analysis is framed, which is almost always a lot less than the detail in an estimate. In the cost of a road, the costs of clearing and grubbing, earthworks, base, sub-base and pavements can all be distinguished. The cost of these elements such as the base can be further subdivided into the costs of extracting stones, crushing them, transporting them, and laying them, and each of those stages can also be broken down. By dealing with components separately – by disaggregating - it is possible to more confidently understand the range of costs each component might have.

While it may seem that the more disaggregation the better, it comes at a cost because many of the disaggregated components will be driven by the same sources of uncertainty as one another. This means that in real life they will be correlated with one another and this needs to be incorporated into a model, which is very

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<sup>11</sup> Asian Development bank (2002) Handbook for Integrating Risk Analysis in the Economic Analysis of Projects

<sup>12</sup> Vose, D (2008) Risk Analysis: A Quantitative Guide, John Wiley and Sons Ltd, Chichester

<sup>13</sup> Metropolis, N & Ulam, S (1949) The Monte Carlo Method, Journal of the American Statistical Association, Vol 44, No 247. (Sep., 1949), pp. 335-341

difficult to do realistically. There is a balance to be struck between the uncertainty that arises when cost elements are summarised to a high level and being able to assess uncertainty more easily as costs are disaggregated.

In broad terms, we find it easier to speak with confidence about the value and uncertainty in individual cost of small elements of a project's cost but we find it harder and harder to build a realistic model that takes account of correlations and other dependencies between these elements the more there are. On the other hand, if a project's cost is lumped into a very small number of items, those items will often be composed of varying proportions of separate materials, trades, services, equipment that will be difficult to disentangle.

There are essentially two ways to deal with correlation and aggregation:

### **1. Limit the disaggregation**

Limiting disaggregation solves the problem of correlation by largely eliminating it. Working, for example, with the total cost of a road means there is no need to worry about the correlation between variables such as the cost of the base and the cost of the wearing surface. The distribution used for the cost of the road as a whole will implicitly include this relationship.

However, as previously discussed, there is a limit to the amount of aggregation that still permits clear judgements to be made about the variability of the cost of a project. Working at a very high level of aggregation will result in very wide range of uncertainty which is not informative for budgetary decision-making purposes. It might also result in an unrealistic assessment of the amount of uncertainty because of the difficulty of thinking about a lot of sources of uncertainty and their interactions all at the same time, especially if the large aggregated parts of the cost share common sources of uncertainty.

### **2. Isolate the sources of uncertainty**

It is specifically recommended that disaggregation of homogeneous variables within which only a small number of uncertainties are at work be limited as much as possible so as to avoid including too much correlation in the analysis<sup>14</sup>. It has been recognised for decades<sup>15</sup> that it is more helpful to think not so much in terms of disaggregating the technological components of a project, but in terms of separating the *sources of uncertainty*. By defining the sources of uncertainty and determining their impact on the different cost elements, the correlation between cost elements caused by sharing a source of uncertainty is included within a risk model in a natural way, often as functional relationships such as between a quantity and a rate that both affect a major part of the total cost.

This isolation of independent sources of uncertainty and determination of how they may affect the cost of a project, in most cases, is the easiest and most rigorous way of avoiding the need for complicated correlations in a model and is usually referred to as either a Risk Factor, or Risk Driver approach.

## **3. Cost risk models**

Common probabilistic cost risk modelling methods can be grouped loosely into three types:

- Line item ranging, which applies a distribution to each line in an estimate or summary of an estimate, usually combined with discrete risk events which are described in terms of the likelihood of them arising and the effect they will have on the cost.

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<sup>14</sup> Asian Development bank (2002) Handbook for Integrating Risk Analysis in the Economic Analysis of Projects

<sup>15</sup> Pouliquen L (1970) Risk Analysis in Project Appraisal World Bank Staff Occasional Papers Number Eleven, John Hopkins Press, Baltimore



- Risk factor/risk driver models in which uncertainties are described in terms of drivers that might each affect several cost elements and where several drivers might act together on a single cost, with a many-to-many relationship between risks and costs.
- Parametric, or hybrid approaches such as a model using a parametric approach for systemic risk impact combined with expected value for project-specific risks.

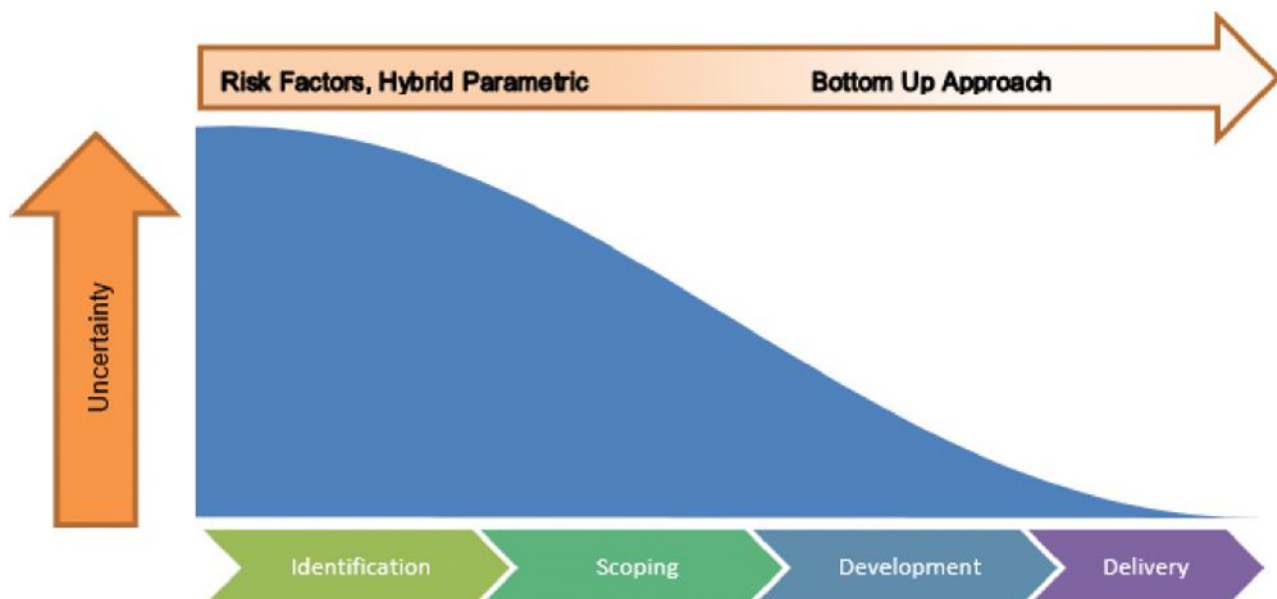
Hybrid methods such as a model using a parametric approach for systemic risk impact combined with expected value for project-specific risks may be regarded as a separate approach again.

A project's cost estimate should be updated during each phase of the project life cycle as project scope is further defined, modified and refined<sup>16</sup>. As the level of scope definition increases, the estimating methods used become more definitive and produce estimates with narrower probabilistic cost distributions. The specific estimating tools and techniques may vary depending upon:

- The general type of risks and specific significant risks associated with a given project.
- The level of definition of scope information available.
- Availability or otherwise of data required to allow certain approaches to be used.
- Skill and/or confidence level of the analyst. Project proponents and/or their contractors are welcome to submit integrated cost-schedule risk analyses as part of funding submissions.

**Figure 3** below illustrates the expectation that a top-down approach is likely to be most appropriate estimation method in the earlier phases (orange gradient), whilst it may be more appropriate to use a bottom-up approach for later estimates as a design is increasingly refined (white gradient). Estimators and analysts should make a judgement based on their individual circumstances.

**Figure 3: Cost estimation method selection**



This guidance note presents several approaches (both top-down and bottom-up) that the department believes are theoretically sound and practical to implement. Worked examples are provided as an Appendix. The approaches are explained in more detail in the supplementary guidance.

<sup>16</sup> AACE (2015) Total Cost Management Framework: An Integrated Approach to Portfolio, Program, and Project Management, AACE International, Morgantown, WV



## 3.1. The preferred approach

The preference is the use of a risk factor methodology wherever practical. Other methodologies/models may become relevant to projects as they progress through their phases and further details have been provided within the **Supplementary Guidance Note to 3A**. The supplementary guidance outlines the following methodologies:

- Risk Factor Methodology
- Risk Driver Methodology
- Hybrid Parametric/Risk Event Model
- First Principles Risk Analysis (FPRA)

The department recognises that line item ranging combined with risk events is a common method of estimating contingency. However, the structural difficulties inherent in line-item ranging make it difficult to arrive at realistic contingency assessments.

## 3.2. Risk factor methodology

This is a top down approach that relies on identifying the major drivers of cost on a project (often related to assumptions about characteristics such as labour productivity, geotechnical conditions etc.) and assigning a factor based on the influence the driver could have on the final cost. If low labour productivity could increase the base cost by 40% it will have a risk factor of 1.4

$$\frac{\text{Base cost} + \text{Variation}}{\text{Base cost}}$$

Identifying risk factors relies on the judgement of experienced project and subject matter experts, to assess the range of uncertainty (risk) a project might face. A useful set of risk factors will be a good fit to the cost as well as the major sources of risk, bridging the two sets of project information. They are the sort of factors that estimators will use to make rapid scenario assessments of the cost differences between options or to assess the impact of late changes to a design or implementation strategy. The risk factor method, assessed using Monte Carlo simulation, has several advantages:

- In most cases it will result in a simpler, cleaner risk model relative to other approaches.
- Analysis generally requires less time than when using line-item estimating.
- Risk factor models can accommodate risks that interact and overlap without introducing conflicting correlations into the model.
- They model the relationship of risk drivers to cost outcomes allowing management to see the connection between a given risk and the potential impact.
- They make it easier to calculate the effect of individual risks on the cost, and then sort the risks by priority.
- They make it simpler to take risks out of the simulation one at a time in order to determine their marginal impact.

At times, it may not be possible to create a set of risk factors that are independent of one another, such as where they depend on the same market conditions, and it might still be necessary to include correlation in a model. One objective of designing a model and selecting risk factors is to minimise or avoid the need to specify correlations. This can usually be achieved and any correlations that remain can be examined by comparing a model with no correlation or full correlation between specific pairs. This can be used to inform

decision making and avoid the need to estimate partial correlations, which is all but impossible to do reliably.<sup>17</sup>

### 3.3. Risk driver methodology

The risk driver method, described by David Hulett, has some of the same features and advantages of the risk factor methodology. It should be noted that risk factor method and the risk driver method, while sometimes seen as being synonymous, have subtle differences. This guidance note interprets a risk driver as per the definition given by Evin Stump<sup>18</sup> as follows:

**A risk driver is any root cause that MAY force a project to have outcomes different to the plan.**

**The risk driver method in its purist form seeks to uncover the causative agents of uncertainty (root causes) which, like the risk factor method, then assigns these drivers to the work elements that they affect. One primary difference between the two methods is that risk factors represent cost drivers that are subject to uncertainty and as such the risk factor method takes a pragmatic approach to the use of inputs that might not be true root causes but do enable engineers and estimators to describe their view of uncertainty associated with a particular aspect of a cost. The other main difference is that the risk driver method, as generally practised, uses the existing risk register to identify the key risk drivers. It should also be noted that risks included in the analysis using the risk driver method should be those at a strategic, rather than technical level.**

### 3.4. Hybrid parametric/risk event model

The premise of this approach is that systemic risks are best quantified using empirically validated parametric modelling. Using appropriate data, the parametric tool provides a cost distribution (optimistic, most likely, pessimistic) representing the aggregate impact of the systemic risks. This distribution is then combined with project specific risks in a Monte Carlo simulation.

Advantages of this approach are:

- It is an easily understood approach with aspects that are already commonly used.
- It employs empiricism and provides a mechanism for learning and continual improvement as more project data is collected.
- Once a parametric model has been developed and validated, using it to quantify systemic risks should be replicable by analysts provided they adopt the same approach to assessing the inputs, thus providing confidence in the model.

The main disadvantages are the challenges and difficulty in obtaining and cleansing project data to build and maintain the parametric model.

### 3.5. First Principles Risk Analysis (FPRA)

The Risk Engineering Society (RES) Contingency Guideline (Version 2, Feb 2019) outlines a bottom-up risk-based approach which, for the purposes of that guideline, is defined as First Principles Risk Analysis (FPRA). The approach aims to capture and validate uncertainties at the lowest meaningful level of Work

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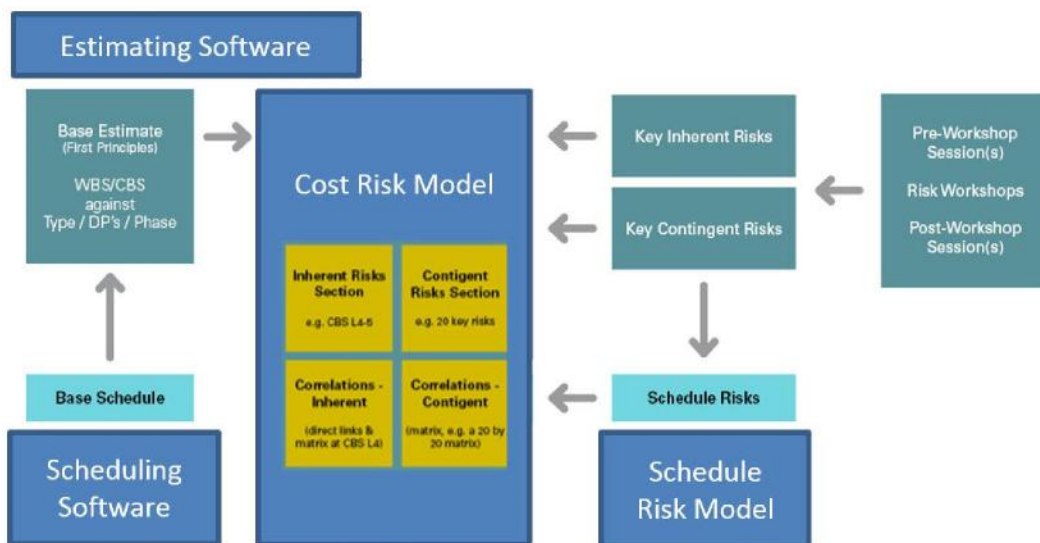
<sup>17</sup> Broadleaf contributed to the technical content of this document and provide further explanation of the risk factor method here <https://broadleaf.com.au/resource-material/the-real-risk-to-your-project-budget/>

<sup>18</sup> Stump, E., (2000), The Risk Driver Impact Approach to Estimation of Cost Risks: Clear Thinking About Project Cost Risk Analysis

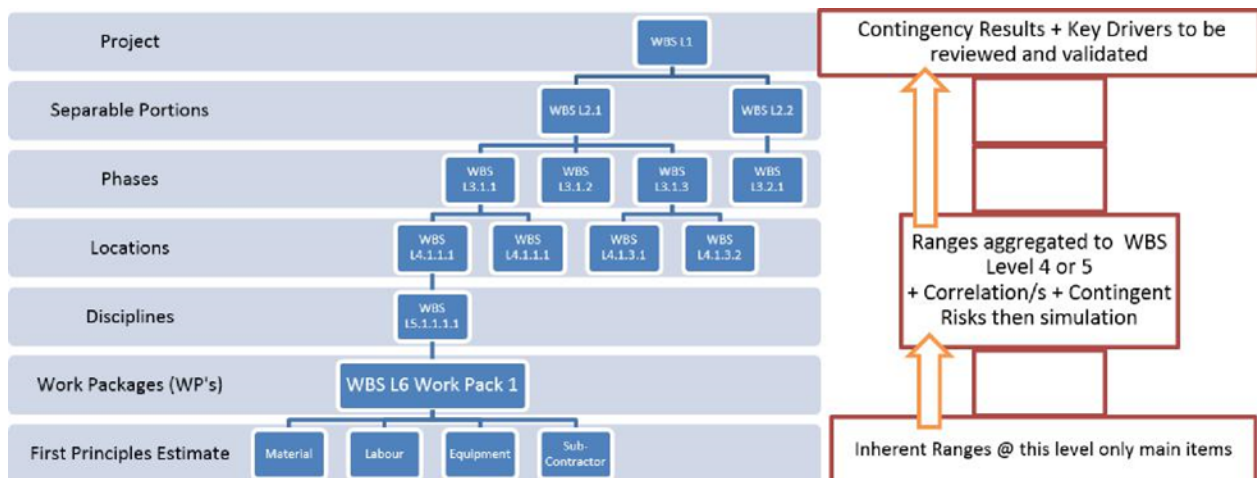
Breakdown Structure (WBS) against appropriate first principle components of cost item (labour, plant, material and subcontract).

The process for the FPRA method is represented in **Figure 4** and **Figure 5** below.

**Figure 4: A typical process map for FPRA methodology – RES Contingency Guideline (2018) - copyright**



**Figure 4: Brief illustration of FPRA flow of information – RES Contingency Guideline (2018) – copyright**



### 3.6. Line item models/3-point estimating

A common method of utilising Monte Carlo simulation to estimate contingency requirements is the technique often described as line item ranging. In this approach the estimate line items or estimate subtotals (by WBS categories or similar) are entered into an Excel spreadsheet which serves as the basis of a probabilistic model. Each fixed line item or subtotal cost entry is then replaced with a statistical distribution of cost outcomes for the line item. These line item distributions are the simulation model inputs.

Placing a distribution on the costs of items in an estimate is an easy extension of standard (base

cost) estimating practice. Modelling large numbers of items with various distributions lends an air of accuracy and sophistication to the model<sup>19</sup>.

Unfortunately the method as generally practiced is highly flawed and in certain situations, particularly when scope is poorly defined, has been found to be less accurate and less calibrated than any other method, including simply relying on a predetermined percentage<sup>20</sup>.

Valid Monte Carlo assessment specifically requires that the dependencies, or correlation, between the model inputs (the model line items) be defined. This is because models used in probabilistic risk assessments take two kinds of inputs in order to produce an output distribution: (1) the marginal distributions (the distributions without regard to the values of the other variables) for the different variables and (2) the dependencies between these variables<sup>21</sup>. While software typically incorporates correlation matrices to facilitate this task, realistic correlation modelling in project risk cost analysis this way is rarely practicable. Most people are unable to make realistic estimates of the amount of correlation between two costs affected by the same source of uncertainty, let alone hundreds of line items.

Further, the task of specifying all the necessary correlations grows combinatorially with the number of variables. A large project may have well over 1,000 cost elements. A 1,000 by 1,000 correlation matrix requires  $(n^2-n)/2$ , or 499,500 correlation values to be determined, at least in principle. Even 50 cost elements give rise to a potential 1225 correlations to be consciously excluded or evaluated.

In practice large numbers of correlations are never addressed directly. More commonly, there will be an attempt to simplify the problem by making assumptions or by aggregating small work elements into larger ones. Unfortunately, these assumptions or consolidations will generally make it much more difficult to assess valid risk distributions, which can destroy the validity of the correlations even if correlations could be assessed reliably. In fact, assessing realistic values of partial correlations is very difficult. Very few people have a reliable understanding of the connection between overlapping dependencies affecting cost items, such as two or more costs driven by varying proportions of several bulk materials, and the partial correlations that will arise in reality, which is what a model must represent. There is no reliable way to assess partial correlations unless they can be measured from data, which is only viable if the project being analysed has the same structure as a large number of prior projects from which data has been saved.

It has been argued<sup>22</sup> that, despite the good intentions of analysts, line item ranging simply captures the team's opinion about the quality of their estimates and does not quantify risks at all. What line item ranging seems to be good at is reliably generating the contingency and accuracy expected by management.

A combined risk analysis/contingency estimating method should start with identifying the risk drivers before the cost impacts of the risk drivers are considered specifically for each driver using stochastic (probabilistic) methods. It is only then that the risk drivers are linked to cost/schedule outcomes. If decision-makers cannot explicitly see the connection between a given risk and the potential impact, then management of the risk during execution will be difficult. Simply putting a range around a line item without considering what is driving the uncertainty provides no insight as to why it may vary from the base estimate and is not recommended practice.

Finally, there should not be a one-to-one correspondence between each element of the model and the corresponding element of the system<sup>23</sup>. One should start with a "simple" model and develop it as needed. Modelling each aspect of the system will rarely be required to make effective decisions, and in fact may

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<sup>19</sup> Broadleaf (2014), Discussion paper: Weaknesses in common project cost risk modelling methods

<sup>20</sup> Burroughs S and Juntima G (2004) AACE International Transactions, Exploring Techniques for Contingency Setting, AACE International, Morgantown, WV

<sup>21</sup> Ferson et al (n.d.) Myths About Correlations and Dependencies and Their Implications for Risk Analysis

<sup>22</sup> Hollmann, J 2016 Project Risk Quantification: A Practitioner's Guide to Realistic Cost and Schedule Risk Management, Probabilistic Publishing, Gainesville, Florida

<sup>23</sup> Law A (2015) Simulation Modelling and Analysis 5th Edition, McGraw Hill Education, New York

obscure important factors. As articulated by British statistician Professor George E.P. Box, “All models are wrong, but some are useful.” In other words, it is not possible to get every detail of the system into a model, but some models are still useful for decision-making.

## 4. Quantitative risk analysis using risk factors

### 4.1. Variations from the base estimate

At the time an estimate is prepared to request approval for a project, design is rarely complete. If

the project is granted approval to proceed, as the design is completed, changes will be made that affect the cost including:

- Some quantities will be increased and others will be reduced.
- Some material and equipment selections will be changed or simply specified more precisely than at the time the estimate was prepared.
- Procurement and construction strategies may be refined or defined in more detail in ways that affect the unit rates for materials, plant, services and labour or the duration of the construction activity.

Even as project execution starts, minor engineering details will still be subject to refinement, information from suppliers will cause changes in plans and the unit rates for materials, plant, services and labour can turn out different from what was expected. Then, during execution, conditions at site, the weather, industrial relations, interactions with neighbouring communities, geotechnical conditions as well as heritage and environmental protection requirements can all result in what is physically done and how much it costs varying from what was assumed in the estimate.

Occasionally, there may be major events that can have a severe effect on a project’s costs. However, few major projects are sanctioned when there are foreseeable potential events with a significant likelihood of happening that could have very large undesirable consequences such as a very large cost increase.

On the rare occasions when work commences in the knowledge that a large additional cost might be incurred, the routine project contingency is unlikely to be used to cover that cost. This is discussed further in [Section 4.4](#). More commonly, a host of smaller events that can affect the cost of a project can be realistically represented as the aggregate effect of all sources of uncertainty on bulk material quantities, rates, durations and other cost drivers.

### 4.2. Common drivers

Research into project cost sensitivity has found that very few projects are subject to significant

uncertainty in more than about twenty basic estimating inputs<sup>24</sup>. Experience with major projects in the infrastructure and resources sectors supports this view. Cost risk models need not be bulky.

The uncertainty in infrastructure cost risk forecasts can almost always be described in terms of the uncertainty in:

- Quantities of materials bought, moved and placed.
- Unit rates for the purchase of materials, equipment or labour.

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<sup>24</sup> AACE International (2008) RP No. 41R-08 Risk Analysis and Contingency Determination Using Range Estimating, AACE International, Morgantown, WV

- Productivity rates for labour and plant.
- Running rates for management, overheads and temporary facilities.
- Lump sum costs of major purchases.
- The duration of the work.

It is always worth considering how and if these uncertainties affect the basic estimating inputs.

It is neither practicable nor necessary to work with the actual physical quantities and rates as there are many detailed instances of them throughout a project. A realistic analysis can be based on an assessment of the extent to which uncertainty about quantities or rates could affect the cost. This reflects the level of definition in an engineering design, the quality of and commitment behind price indications taken from the market and past work, and the maturity of major design or strategic decisions.

If the quantity of concrete in a slab increases by 10%, and cost is estimated on a volumetric basis that includes supply and placement, it is reasonable to suppose that the cost will increase by 10% as well. If the quantity is subject to  $\pm 5\%$  variation, then the cost will be too. It is only necessary to focus on how much uncertainty about quantities or rates could affect parts of the cost rather than on how much the value of quantities or rates might vary in themselves.

This is the approach an estimator will often adopt if asked to carry out a what-if analysis. The effect of lengthening a noise wall on a road can be assessed relatively quickly, without going back to first principles, by identifying the cost items associated with the wall and factoring them by the proportional increase in the length of the wall. Where there is insufficient time or an approximate answer will be enough, this is a cost-effective practice. It provides sound information at the least cost.

There is no formula for choosing the most useful risk factors to represent uncertainty in a cost estimate. The selection of factors can change from one job to another and from early stages of design through to project execution. The most useful way to decide on the factors to use for a particular job is to start at the highest level, ask whether it is possible to describe the uncertainty in the major cost drivers at that level and if not then break it down a level and try again. The question “Why should we add more detail here?” is a good test. Extra detail will always add to the effort required for the analysis, the difficulty of understanding the outcome and the challenge of communicating it to decision makers or reviewers. There should be a good reason for making a model increasingly granular.

The way the costs are broken down depends on how the team understand the nature of the uncertainty affecting the cost. Some common structures are set out in **Table 1** showing how costs might be broken down for the purposes of assessing uncertainties and the cost categories to which they can apply. The factors refer to how much uncertainty there is associated with something that affects the cost, not to the thing itself.

**Table 1: Risk factor structures**

Factor type	Possible cost categories to which the factor applies
Quantity uncertainty	Physical areas of a project; Disciplines (bulk earthworks, detailed earthworks, in-situ concrete, precast concrete, pavement, rail, structural steel, electrical ...) Major structures (road, bridges, culverts, viaducts, gantries ...)
Rate uncertainty	Bulk materials as a whole, labour, plant, freight, subcontractors' distributables; As above but breaking bulk materials into earthworks, concrete, steel, cabling and other categories; A subcontracting based structure where major parts of the work are expected to be subcontracted and the estimate represents the price it is expected will be struck for each piece of work.

Factor type	Possible cost categories to which the factor applies
Labour Rate uncertainty	Project as a whole; Discipline based; Location based.
Productivity uncertainty (generally broken down in the same way as the Labour Rate factor)	Project as a whole; Discipline based; Location based; Greenfield and brownfield portions.
Management overhead running rate uncertainty	Project as a whole; Engineering, Procurement and Construction Management (EPCM) and Owner's team; Project office and site office.
Lump sum purchase uncertainty	Project as a whole, reflecting a degree of confidence in budgetary quotes and market information; Equipment types; Suppliers; Long lead, normal orders and off the shelf.
Duration uncertainty (linked to representation of overheads)	Project as a whole; Prior to approval to proceed, execution phase, commissioning; Major phases in staged works.

These are only examples but they cover the vast majority of work carried out using this approach and they illustrate the principle of breaking work into parts with common characteristics from the top down. A project might use some of these, all of them or a completely different set of categories. Examples of some can be seen in the sample probabilistic risk factor models accompanying this guidance note.

The key principle when selecting risk factors is to reflect the way uncertainty in the estimate is understood by the personnel responsible for the design, estimate and plans. If uncertainty about a quantity or rate is markedly different from one part of the estimate to another, and both can have a significant effect on the cost, then it may be worth separating them in the model. If the uncertainty in the quantities or unit rates for one bulk material is affected by different matters to those affecting another bulk material, it may be worth splitting these up. If there is uncertainty about productivity, it is worth separating, in the model, the costs related to labour hours such as labour cost, plant cost, supervisions and subcontractor's distributables so that the productivity uncertainty can be applied to those parts of the cost. Of course, if parts of the project have unusual characteristics, quite different from the rest of the work, but are so small that they will never have a significant effect on the total cost, there is no point splitting them out.

If costs such as plant, accommodation and supervision are wrapped into the labour rate, they might not have to be separated from one another as they will all be driven by labour hours, which is in turn driven by quantity and productivity uncertainty. However, it is difficult to model the effect of productivity uncertainty on costs that consist of a mix of bulk material and labour related categories so it is usually desirable to separate these two types of costs.

Appendix A provides further detail on the level of detail to include in a model and methods to derive relevant risk factors on projects.



### 4.3. Risk events

The risk factor approach described in this guidance note avoids the practical problems of dealing with a large number of interacting events or a large number of correlated line-item variations by focusing on the major estimating inputs that the events will affect and which drive the individual line items. It draws on the experience and judgement of project personnel to understand the potential variation in these cost drivers, which are the major inputs to the estimate.

Experience shows that risk factors, such as those outlined in **Table 2**, provide a concise and effective way to describe the potential variation of project costs.

Representing the uncertainty in a project's cost directly in terms of these high level cost drivers avoids having a large number of separate cost distributions individually affected by the one underlying source and so having to be linked by poorly understood correlations, as is usually the case with line item risk models. Wrapping up in a few straightforward risk factors the various small and medium scale risk events that could affect the quantities of material, unit rates, labour productivity and other important cost drivers, makes it easy to understand any interactions that do arise and model them. This is in contrast to many risk register based models with a large number of individual risk events that might have complex interactions, which are not examined or taken into account. As noted earlier, projects are rarely approved with very large risks that have a realistic chance of affecting them. In the rare cases where this does happen, separate financial provision will usually be made rather than trying to wrap them into the general contingency.

Individual risk events can be combined with a driver-based approach but experience shows that, once a model is constructed around high level cost drivers and the uncertainty in them, the need for a large number of separate small and medium sized event risks falls away. Some examples of items that are routinely modelled as separate items and the risk factors that are usually seen as encompassing their effects, representing uncertainty in cost drivers, are listed in **Table 2**. This is just an illustrative selection.

**Table 2: Examples of risks and relationships to risk factors**

Detailed risk (examples)	Risk factor	Costs affected
Crib rooms have to be located further away from work front than assumed; Site access constraints; Quality of supervision differs from assumptions; Site works poorly co-ordinated leading to congestion.	Labour productivity	Labour Plant and anything else driven by labour hours
Low cost contractor or supplier stops trading; Validity on quotes expire before approval; Subcontract market becomes more heated.	Subcontractor rates Bulk material supply costs	Subcontract Bulk material supply costs
Vendor data changes when firm orders are placed; Detailed design alters equipment selection.	Equipment rates	Equipment cost
Need to shift ventilation shaft locations; Tunnel portal detailed design outcome; Traffic data from later surveys and effect on detailed design; Accuracy of survey of interchanges with local roads.	Bulk material quantities	Bulk material supply costs Labour costs associated with bulk materials and anything driven by labour hours
More rock than assumed must be excavated; Rock is not rippable; Intensity of ground stabilisation required;	Bulk earthworks rate	Bulk earthworks cost



Detailed risk (examples)	Risk factor	Costs affected
Proportion of acid forming soils differs from assumption.		
Precast structures supplier fails to meet milestones forcing resort to alternative supplier	Precast concrete rate	Precast concrete cost
Unable to site concrete batch plant in preferred location; Have to use quarry that is further away than planned.	Bulk concrete rate	Concrete cost
Activity in market for road headers, tunnel-boring machines (TBMs) or other large equipment increases.	Major equipment cost rate	Purchase prices of road headers, TBMs or other large machinery
Driven pile resistance depth differs from assumption, on average; Bored piles encounter soft ground at depth.	Pile bulk material quantity	Pile supply costs Labour costs associated with piling and anything driven by labour hours
More stringent corrosion resistance standard imposed on piles.	Pile bulk material rate	Pile supply costs
Delays obtaining responses from neighbouring rail operator; Delays arriving at agreement with Telstra about cable relocation; IR problems on site; More or less wet weather than assumed; Plant reliability different to assumed; Actual productivity on site different to what has been assumed.	Duration (overall or key stages)	Time related costs
Client interference; Professional rates come under market pressure; Documentation requirements become more onerous than expected so need additional personnel.	Site office running rate (\$/mth)	Site office costs (also affected by duration)

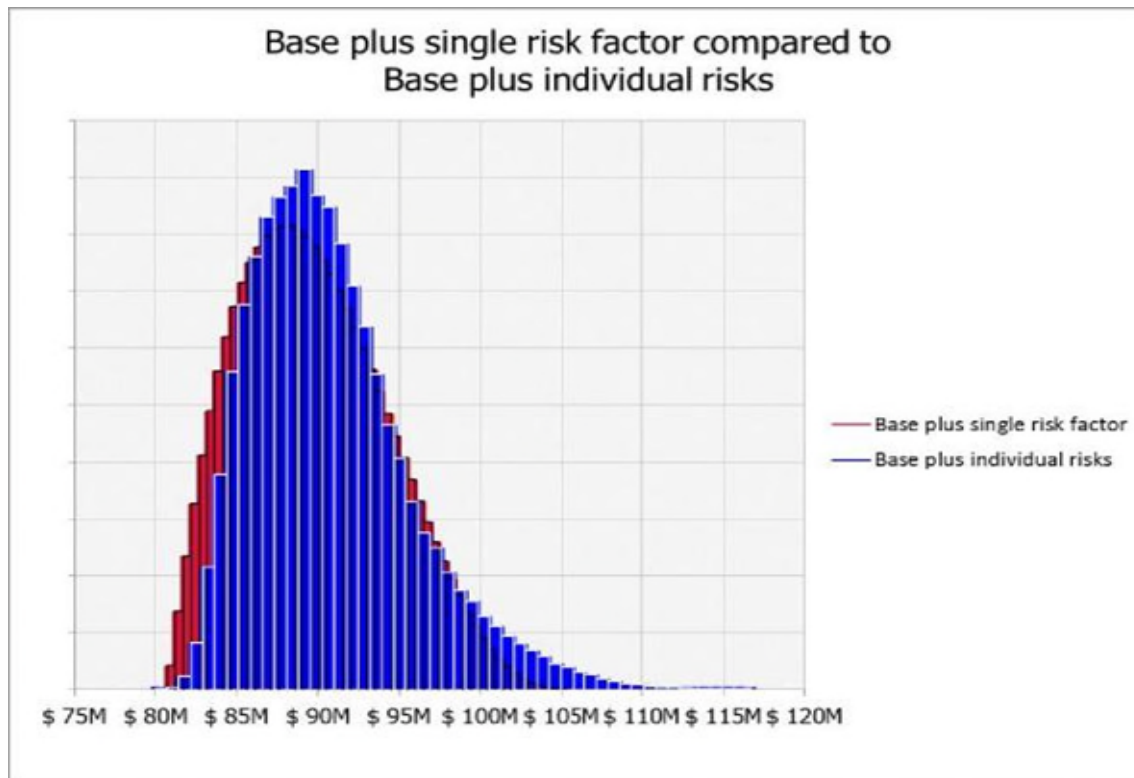
There are two reasons why multiple event risks can be modelled realistically using high level factors represented by continuous distributions:

- Most of the uncertainties described in the event risks have a range of possible consequences, best described by continuous distributions. Many have no uncertainty associated with them occurring or not because they are not discrete events and there is a 100% chance of them causing some deviation from the base estimate.
- The combined effect of even a small number of individual distributions generates a fairly smooth continuous distribution. Personnel with experience of similar work will understand the net effect of the uncertainties and can describe it, subject to taking care about the biases discussed in [Section 5.2](#).

This is illustrated in **Figure 6** which shows an overlay of the same hypothetical project modelled in two different ways. The blue output shows the result of modelling the project with a base cost of \$80m and with ten risk events as per **Table 3** modelled separately. The red output shows the result of modelling the same project with a base cost of \$80m but with a single risk factor representing the aggregate impact of the ten individual event risks. The risk factor is defined by optimistic, likely and pessimistic parameters of 5%, 10% and

20% variation in the base cost estimate using a Triangle distribution. The individual risks' three point estimates (P10, Mode and P90) and event probability values are shown in **Table 3**.

**Figure 5: Overall risk factor compared to multiple events**



**Table 3: Individual risk events used in illustration**

Risk	Optimistic	Likely	Pessimistic	Probability
A	\$0.4M	\$1.3M	\$2.1M	100%
B	\$0.1M	\$0.9M	\$2.7M	50%
C	\$1.0M	\$2.7M	\$7.1M	75%
D	-\$1.0M	\$0.0M	\$2.5M	10%
E	\$0.3M	\$0.7M	\$1.3M	25%
F	\$0.0M	\$0.6M	\$1.6M	100%
G	-\$0.2M	\$1.3M	\$5.3M	40%
H	\$1.5M	\$2.0M	\$3.2M	100%
I	\$1.2M	\$2.0M	\$3.6M	10%
J	\$2.3M	\$5.3M	\$12.4M	20%

The significance of this illustration is not the specific details of the risk events or the parameters of the single overall risk factor, which are hypothetical although not unrealistic. It is the fact that a set of detailed items generates broadly the same type of outcome as is represented by a single factor intended to cover their aggregate impact on a base cost estimate. So long as a team feel that they understand the work well enough

to assess the aggregate effect of several sources of uncertainty, based on their experience with similar projects, they need not be concerned that they are doing so. It is a perfectly reasonable approach.

This illustration uses just one factor. A real analysis will have several and the same principle applies. The behaviour we understand for a whole project's cost risk can be described realistically using risk factors. Risk events are not necessary and have many drawbacks as the basis of a model, as discussed earlier.

If a team does not feel comfortable assessing uncertainty at a high level, they might seek to break the cost into smaller parts and apply the same approach to those parts. These might be regions of the work subject to different sorts of risks. This is only worthwhile if the personnel concerned feel more confident about the assessments they can make at the detailed level and are sure that they have captured all the detail necessary for a realistic assessment of the overall uncertainty. If not, there is no point building a more granular model.

There is no rule about where to stop breaking the analysis into ever smaller parts. Experience with this method shows that the urge to introduce greater detail, beyond the point where it relates to parts of the project having different risk characteristics, hardly ever makes a material difference to the outcome but it does absorb an appreciable amount of additional effort, so long as each assessment is diligent and takes appropriate steps to avoid bias. Only a few high level factors are generally required to model uncertainty in an estimate. This is sufficient to allow a professional team to consider and describe the uncertainty they can see arising in an estimate.

As previously mentioned, in most circumstances, it has been found only twenty or so factors are required to describe the uncertainty in a project so this method results in a more compact model than the alternatives in common use. It absorbs less effort and produces greater understanding. Uncertainty factors do interact with one another but the interactions are generally straightforward, such as between quantity and unit rate variations or between duration and temporary facilities cost variation, in each case the product of the two factors represents their combined effect, and they can be built into a model relatively easily, usually as simple functional relationships.

## 4.4. Uncertainty that should not be captured

Few major projects are sanctioned when there are foreseeable events with a significant likelihood of happening that could have very large undesirable consequences such as a very large cost increase. Those who provide the funding will not usually sanction a project that they believe has an appreciable prospect of incurring catastrophic additional costs.

Circumstances in which projects might commence in the knowledge that a high impact event could occur may include:

- Work to reduce a serious threat, such as emergency flood mitigation, stabilisation of collapsing infrastructure or reinstatement of a critical washed out transport route; and/or
- Projects constrained by absolutely fixed dates such as hosting the Olympic Games or similar situations.

If there is a strategic need to embark on work that could be subject to a very large cost increase, this is best managed as a stand-alone contingent funding requirement with an agreed trigger and controls on the release of the funds. To incorporate it into a general project contingency only serves to hide the nature of the requirement and obscure the special character of the costs involved. It is a special requirement and will be best managed apart from the general funding of a project.

Using a weighted impact to calculate the contingency required for an event such as this is rarely satisfactory. If there is a potential event that could cost \$100 million and it is thought to have a 20% chance of happening, holding \$20 million (20% of \$100 million) will not help. If the event occurs, \$100 million will be needed. Weighting risks by their likelihood of occurring only works when there are many small or medium events to be covered and they are independent of one another. The way contingency funds are assessed and managed for a large number of independent small and medium scale risks is not suited to making provision for very high impact risks with an appreciable probability of occurring.

Earthquakes, terrorist attacks, fire destroying a yard full of earthmoving machinery, a contagious disease being brought in by one worker and causing closure of a site for a few weeks are all conceivable but very unlikely. These are generally regarded as normal risks associated with what is sometimes called 'business as usual'. There may be insurance held against some of these but some will simply be accepted due to their rarity.

In addition to excluding extreme events, attempting to capture every one of the uncontrollable events, such as those in the previous paragraph, which could impact on the final cost of a project can become an unproductive exercise. It is impossible to conceive of the multitude of smaller events that may occur or of how they might overlap or interact. The majority of these smaller risk events will be covered by one or more major risk drivers. Identifying the key risk drivers and their uncertainty, the range of each one and the likelihood of values within that range, before linking the risk drivers to cost outcomes, is a far more efficient and theoretically sound approach to quantifying project-specific risks.

There are numerous types and sources of risk. Cost risk analyses attempt to address some, but not all these risks. Those typically excluded are extreme events such as the cost consequences of an earthquake occurring. There are good reasons to leave out some of these; cost risk analysis is intended to provide decision makers with information to help them successfully manage projects and the inclusion of extremely rare events with large impacts will not aid decision makers with project budgeting. This exclusion of some risks is advisable and only including those factors impacting management's decisions is appropriate for a project estimate. Rare events might be considered but it does not help to blend them into the general project contingency.

### **Expected Value and Risk**

The following example demonstrates why using weighted impact to calculate contingency requirements for a large event risk is rarely appropriate.

The notion that expected value (probability multiplied by consequence) is the way that rational people make decisions appeared long before modern economic theory. In the early eighteenth century Daniel Bernoulli systematically attacked the idea and introduced his concept of utility whereby there is no reason to assume that the risks anticipated by each individual must be deemed equal in value; it all depends on individual circumstance<sup>25</sup>. For example, say you are offered the chance to make a wager where you have a 90% chance of winning \$1 million but if you lose, you have to pay \$100,000. The expected value is thus  $(\$-100,000 * 0.1) + (\$1,000,000 * 0.9) = \$890,000$ .

Surely, it would be foolish not to take this offer? After all, you expect to pocket \$890,000 from the transaction. The subtlety is that expected value is the long-run average of repetitions of the experiment it represents. In the binary case of the wager just described, \$890,000 isn't even a possibility and the problem is that, for most people, \$100,000 is likely to be a catastrophic loss they are unlikely to afford. Similarly, using the expected value (weighted impact) to calculate the required contingency for a potentially catastrophic event and then believing that the risk has been accounted for is highly misleading.

The same sort of reasoning explains why we regularly pay premiums to an insurance company when the mathematical probabilities indicate that we will lose money; the premiums we pay exceed the expected value of events such as our house burning down or our car being stolen. We enter into these losing propositions because we cannot afford to take the risk of losing our home or car. We prefer the gamble that has 100% chance of small loss (the premium) rather than pocket this small saving because a disaster would be catastrophically ruinous if it were to occur.

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<sup>25</sup> Bernstein, P., (1998) *Against the Gods: The Remarkable Story of Risk*, Wiley

## 5. Risk workshops and eliciting expert opinion

### 5.1. Overview

Once the risk factors and other values required for the model have been identified, values must be assessed. It is rare to have representative historical information that can be used without any human intervention; risk analysis models almost invariably involve some element of subjective estimation. Obtaining data from which to determine the uncertainty of all of the variables within the model, so that subjective assessments are not needed, is usually not possible because:

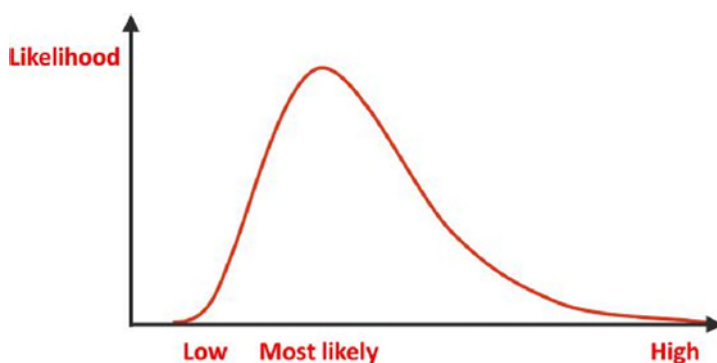
- The data have never been collected in the past.
- The data are too expensive to obtain.
- Past data are no longer relevant, or decision makers do not believe they are relevant.
- The data are sparse.

The lack of data means that subjective estimates must be made regarding the uncertainty of the variables within the model. Subjective assessments will always be at risk of bias. While we have no option but to use them, it is prudent to take steps to limit or avoid bias. Techniques to overcome potential bias are discussed throughout this section.

If risk events are significant enough to be described individually in a model, the probability of each one occurring will have to be assessed. However, the majority of the assessments required in risk factor models are for continuous distribution functions, as illustrated in **Figure 7**. The discussion here is mainly about assessing distributions or continuous variation.

The distribution used represents what those who produce it believe about the value it represents. It is not an engineering measurement or data derived from scientific observations. It is an informed opinion.

Distribution parameters are usually assessed in terms of three point estimates of low, most likely and high possible outcomes for the value concerned. There are problems with using minimum and maximum values to describe the low and high ends of the range so, as is explained in [Section 5.4](#), upper and lower percentiles, usually P10 and P90, are often used to describe the spread of a distribution.



**Figure 6: Risk factor distribution**

The problem with using minima and maxima to specify the upper and lower extent of a distribution is that absolute extreme values are very difficult to contemplate because they represent circumstances that occur very rarely. Assessments of minimum and maximum values can be influenced fairly easily by a facilitator, which means that they are not reliable modelling inputs. P10 and P90 assessments are more robust. P10 and P90 values represent circumstances most people will encounter, something that will affect one job in ten.

In order to avoid many pitfalls that can interfere with obtaining realistic assessment of uncertainty, it is important to understand a little about how the process can go awry and to use a sound approach to avoid

this. In particular, the common practice of assigning values to potential variations first and recording the rationale for the values afterwards, or not recording the rationale at all, is not recommended.

## 5.2. Biases

There are many ways to analyse and describe the way human assessments can be affected by unconscious and deliberate bias. This has been the subject of considerable academic research for decades<sup>26</sup>.

**Table 4** outlines some of the forms of bias that are common in the assessment of uncertainty for project cost contingency analysis. These effects can be described in various ways so other discussions of bias might use different terms but they cover the same concerns.

**Table 4: Biases**

Type	Outline description
Optimism	There may be many reasons for a project team to want to see a project accepted for implementation and some people simply prefer to adopt an optimistic view of the future. This can lead to understating the base estimate and the amount of variation that might arise, especially when assessing variations that will tend to drive up cost.
Availability	People make assessments based on the information they have to hand. They will rarely take deliberate steps to seek out information from other settings that might conflict with that which is most readily available. This can lead to the potential for major deviations from planned costs to be overlooked or set aside even when evidence from other areas would raise serious concerns.
Confidence	Most people involved in projects believe in their ability to deliver projects. Training and culture generally encourage a positive attitude. This is at odds with an objective assessment of the real prospects of a piece of work being delivered to plan.
Anchoring	Once a cost or a key parameter such as a bulk quantity, a rate or productivity factor, has been declared, assessments of possible variation from that value will tend to remain close to that starting point. This results in a systematic tendency to understate the extent to which actual outcomes could differ from the assumptions incorporated into an estimate whether they will raise or lower the cost. The assessment is anchored on the base estimate.

All these and other forms of bias can completely undermine the realism of a quantitative risk analysis. It is important to take deliberate steps to overcome them. Two straightforward strategies can be used to improve the realism of uncertainty assessments:

- Selecting a competent team for the assessment.
- Using a process that will help avoid bias.

<sup>26</sup> Spetzler and Stael von Holstein (1975) Probability Encoding in Decision Analysis. Management Science 22(3), 340-58

### Feelings influence risk perception

People's risk perception is influenced disproportionately by many things, including the rarity of the event, how much control they believe they have, the adverseness of the outcomes, and whether undertaking the risk is voluntary or not. For example, people in the United States underestimate the risks associated with having a handgun at home by 100-fold, and overestimate the risks of living close to a nuclear reactor by 10-fold<sup>27</sup>.

## 5.3. Participation in assessment

Participants in risk assessments can be self-selecting or sometimes unwilling conscripts. **Table 5** sets out a checklist of points to bear in mind when deciding who to involve the assessment of uncertainty factors and probabilities for a contingency model. It is almost always a bad idea to have a single individual assess the ranges of values used for a risk model, even if their work is reviewed later by others. A workshop process, preferably facilitated by someone independent of the project, is almost always preferred. The assessment should involve participants challenging one another so that tacit assumptions that might be incorrect and gaps in information are exposed and resolved.

**Table 5: Participant selection**

Issue	Description
Conflict	If any major differences of opinion or outright conflicts about the work are known in advance, they can be addressed in a risk workshop but it is usually far better to deal with them beforehand in the interests of using the participants' time efficiently.
Diversity	An individual working alone will find it difficult to avoid bias creeping into their assessments and a team of people who are all of one mind will be little better. Some teams are used to challenging one another but even they can find themselves caught in entrained patterns of thought and shared assumptions. Including people from outside the team in the process is invaluable. They might be people working on similar projects elsewhere in the same organisation, experienced personnel who have retired and can be brought back as consultants or external experts brought in just for the risk analysis.
Numbers	Three or fewer people will rarely generate the interactions necessary to overcome their biases even if they are reasonably diverse. On the other hand, subject to their familiarity with the process, workshops with more than about fifteen people tend to fragment and become inefficient as side conversations crop up and attention wanders. Between five and twelve seems to be an effective team size.
Dominance	If there is one person in the workshop to whom the others will defer, a senior manager or dominant character, this will stifle input from the others. Ideally, everyone in the workshop should feel equally entitled to talk and be willing to do so. Well informed senior managers generally understand this and will leave

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<sup>27</sup> Sutherland, W.J., Spiegelhalter, D. & Burgman, M.A. (2013) Twenty Tips for Interpreting Scientific Claims. Nature 503, 335-337



Issue	Description
	their personnel to carry out the process by themselves and perhaps review the output later.
Comprehensiveness	It is important to have someone present to speak on all the key parts of a project, otherwise some items might be poorly assessed. Participants will usually include someone familiar with the estimate, the schedule, the construction strategy, procurement, environmental and community relations matters, and the engineering design. Commercial and contractual matters might play a role but these are usually addressed separately once the cost uncertainty is understood except where alliancing or subcontracting is a source of concern.
Commercial sensitivity	<p>If the engineers developing a project at the study stage have been engaged by a proponent through a contractor that will be bidding to implement the work if it is approved to proceed, there may be a conflict of interest for the engineers. Expectations they set and uncertainties they acknowledge in the workshop might be used as negotiating points when the proponent is engaging a contractor to deliver the work resulting in biased assessments. For instance, if the engineers agree that there is some chance of detailed design reducing the scale of bulk earthworks and so reducing the estimate, the proponent might use this to suggest that the estimate is padded when negotiating a contract. There is no simple resolution to this dilemma although a mature approach from all parties should enable the exercise to be successful.</p> <p>Where a proponent has in-house expertise, they might carry out the risk assessment with their contractor and then review it in-house later.</p>

Assembling a sound team for a risk assessment is not difficult but it is easy to get it wrong and undermine the realism of the process. A little effort devoted to selecting a suitable team, especially ensuring some independent input, will pay off in the quality of the outcome. The modest cost of engaging external subject matter experts to provide an independent voice on key areas of a project may well be a worthwhile investment.

## 5.4. Assessment process

### Overview

This section describes a process for assessing a single input to a model. It is based on assessing the range of a distribution representing the possible variation in an uncertain quantity, unit rate, lump sum or other continuous variable. The same approach is valid for assessing event probabilities with the obvious difference that only a single value has to be estimated. The approach has two main parts:

- Establishing the context of the assessment, which includes exploring possible variations that are described in qualitative terms.
- Assessing quantitative measures of a possible variation.

Risk factors are usually described in terms of percentage changes from the values assumed in the estimate. To avoid becoming caught up in discussion about the fine details of the estimate, which might not always be very informative, it can be useful to think of variations in terms of the percentage change in the relevant costs that the factor could cause rather than the percentage change in the quantity the factor describes. This draws



attention to the fact that there may be parts of a cost that will be more volatile than others and the overall uncertainty in the whole of the relevant cost will be somewhat lower than that in the most volatile parts.

## Establishing the context

The purpose of this stage is to establish the foundation upon which the variation is being assessed, to understand how the factor being assessed could turn out differently to what has been assumed, and to bring all the participants to a shared view of this contextual information. It also produces a valuable summary of the risk to a project in relatively high level terms. This can be extremely useful for communicating the project's risk profile to senior management and others.

A well tried method for establishing the context is to work through the steps set out in **Table 6**, adhering strictly to the sequence in the table. Each entry will typically consist of anything from a couple of points to a short paragraph.

**Table 6: Establishing the context**

Step	Description
Assumptions	Note any assumptions that are important to the factor being assessed and might not be obvious to someone who understands the sort of work being proposed in the project but who has not been party to the development of the estimate or the risk assessment.
Status of work to date	Describe how the costs affected by this factor have been estimated. It might have been built up from first principles, factored from related jobs, based on a subcontractor's quotation or derived by other means or a mixture of methods.
Sources of uncertainty	A description, without any attempt at quantification, of what could cause the outcome to differ from the estimating assumptions.
Pessimistic scenario	A brief description of what would be happening if the factor was to turn out such as to result in higher costs.
Optimistic scenario	A brief description of what would be happening if the factor was to turn out such as to result in lower costs.
Likely scenario	A brief description of how the matters used to describe the pessimistic and optimistic scenarios are expected to turn out.

Examples of typical context statements are shown in **Table 7** and **Table 8**.

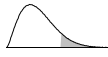
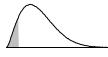
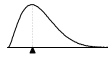
**Table 7: Quantity factor context**

Data table 1: Bulk material quantity - Earthworks	
Date	Participants
<b>Assumptions</b>  Shallow fall, 1:50, across the site  About 25% covered in topsoil to 200mm depth Preliminary geotech proves to be characteristic of the whole site	

Data table 1: Bulk material quantity - Earthworks	
Date	Participants
<b>Status of work to date</b>  Topographical survey obtained and 3D model used to establish levels for a design  Initial geotech investigation undertaken Earthworks design about 70% complete	
<b>Sources of uncertainty</b>  Size of culverts in creek crossing, subject to hydrological modelling Batter angles in northern section  Top soil depth	
<b>Pessimistic scenario</b>  Need up to two more culverts in the creek crossing, possibly with increased section  Top soil depth up to half a metre on one third of the site Batters have to be laid back along the northern section	
<b>Optimistic scenario</b>  Able to reduce culvert sections a little Minor increase in batter angles in the southern section	
<b>Likely scenario</b>  As assumed in the estimate	

**Table 8: Rate factor context**

Data table 2: Temporary facilities supply rate	
Date	Participants
<b>Assumptions</b>  Based on an estimated rate for site offices, services and light vehicles taken from previous project	
<b>Status of work to date</b>  Budget quotes obtained from several suppliers  Used average with some adjustments	
<b>Sources of uncertainty</b>	

Data table 2: Temporary facilities supply rate	
Date	Participants
Supervision levels required to run the project	
Quality and competitiveness of market for facilities and services	
<b>Pessimistic scenario description</b>  Supervision levels have to be increased with additional engineering support Low cost tenderers cease trading	
<b>Optimistic scenario description</b>  Do not need as many light vehicles as has been assumed	
<b>Likely scenario description</b>  As estimated	

## Quantification

Establishing the context increases the chances that the participants will take account of all relevant information and offers all a chance to arrive at a common understanding of the risk factor being assessed. It helps to address biases arising from limited information and unrealistic optimism or pessimism. When it comes to assessing the quantitative variation associated with a risk factor, there is one remaining bias to be addressed, which is anchoring bias.

If people are led from the existing situation to consider how much worse and better it might become, they will be anchored to the starting point. This will result in unrealistically narrow ranges of outcomes being forecast. The second stage of the assessment is designed to overcome this tendency.

To reduce the chance of assessments being anchored near the base estimate values, it is recommended that the assessment proceed by the steps set out in **Table 9**. Strict adherence to the order of these steps is crucial. It is important to note, and to make clear to the participants, that the worst and best case assessments are important parts of the process, to break the anchoring effect, but they are not used in the model. It is important that they are both plausible and extreme so that they free participants to contemplate realistic levels of variation for the values that are used in the model, the P10, most likely and P90 values.

**Table 9: Quantitative assessment process**

Step	Guide
Worst case	Consider the worst variation that could be explained under current circumstances if luck went against the project and controls failed to operate effectively. This should be an assessment of an outcome that no one ever expects to see but could be explained in principle, so extreme that it is impossible to envisage anything worse. It should feel uncomfortable to contemplate this value.
Best case	Consider the absolute best case that could ever be imagined. This will represent a technically feasible but extremely optimistic outcome, not

Step	Guide
	unrealistic in the sense that it could happen in principle but so optimistic that no one would ever expect it to happen in practice.
Pessimistic (1:10)	Having used the worst and best cases to open up the team's thinking, now ask for a realistically pessimistic view of the outcome. This will be something that could arise and might be within the experience of some of the participants. They would be disappointed if it was to happen but would not regard it as completely extraordinary. This might correspond to a one in ten chance, something that one in ten similar projects could encounter – a level of cover that would give a project or work package manager a sense that they have a 90% level of confidence of this estimate input being sufficient.
Optimistic (1:10)	In the same way as for the pessimistic assessment, ask about an optimistic outcome that would not be entirely extraordinary but would be very gratifying. This should represent a combination of good luck and good management but not be beyond belief. It should be an outcome that could be contemplated in about one in ten similar projects and might warrant recognition as a very good outcome.
Likely	The forecast of the most likely level of variation from the estimate will often be zero but there are circumstances in which it might not. If the estimate has been prepared under policies that are strictly lean or deliberately conservative, it might be expected that the most likely outcome is not equal to the base estimate. Whether any non-zero most likely variations are later absorbed into the estimate or not is a matter of policy for those concerned but, if it is, the pessimistic and optimistic variations should then be reviewed.

The sequence in which these five values are assessed is very important. It works in from the extremes to the centre and addresses the pessimistic side of the variation before the optimistic side at each stage as illustrated in **Figure 8** where the numbering shows the sequence in which the points are addressed.

**Figure 7: Assessment of quantitative variation**



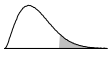
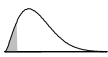
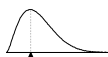
This approach helps to overcome anchoring. The pessimistic and optimistic assessments are used as the P90 and P10 values of the risk factor distribution and the most likely is used to define its mode.

## Documenting the assessment

It typically takes an average across the duration of a workshop of about ten to fifteen minutes to complete each factor's assessment. The first few assessments will usually take longer and teams who are familiar with the process will proceed more swiftly than those for whom it is new.

If the deliberations of the team are recorded as the workshop proceeds, the assessment will be largely documented by the time the workshop ends. The form shown in **Table 10** has been found to be a cost-effective means of both enforcing the sequence on which the process depends and capturing information as the workshop proceeds.

**Table 10: Data table format**

Date	Participants	
<b>Assumptions</b>		
Information a third party would need to understand the rest of the assessment		
<b>Status of work to date</b>		
What underpins the estimate and assessment below?		
<b>Sources of uncertainty</b>		
What underpins the assessment below?		
<b>Pessimistic scenario description</b>		
Summary of conditions that would be seen if things went poorly		
<b>Optimistic scenario description</b>		
Summary of conditions that would be seen if things went well		
<b>Likely scenario description</b>		
Summary of conditions actually anticipated		
<b>Range estimate</b>		
Scenario	Forecast	notes
Worst		<i>As bad as it can get without the job being cancelled</i>
Best		<i>Best anyone could conceive of - textbook case</i>
Pessimistic ( $1/10$ )		<i>These three are used in the model, the two above are part of the elicitation process</i>
Optimistic ( $1/10$ )		
Likely		

These data tables encapsulate the uncertainty associated with a project in a form that is readily communicated and forms a valuable foundation for later reviews and updates.

# 6. Model structure

## 6.1. Design principles

A project cost risk model must connect sources of uncertainty to the estimate in a way that allows their aggregate effect to be assessed. This involves deciding how to describe uncertainty or risks, how to summarise the estimate and how to link one to the other. This part of the exercise is often overlooked with a default model structure being used without consideration of how these three components are designed: the way risk is described, the way the estimate is summarised and the way the two are connected. Attention to this aspect of the analysis can improve its realism and avoid unnecessary effort.

The key design principles recommended here are:

- A model should be as simple as it can be while including enough detail to represent the material uncertainties affecting the cost.
- The relationship between real world uncertainties and elements of the model should be as straightforward and direct as possible.

Before explaining a methodical approach to developing a model structure, it is useful to remember why there is uncertainty in a cost estimate so that attention remains focused on the uncertainty being analysed rather than on the technical details of models, which are only a means to an end not an end in their own right.

Uncertainty arises from the fact that it is impractical to develop and prove a design in advance of construction to the point at which it could be built with no further changes. At the stage when a risk analysis is undertaken, in general, the level of definition will be more or less uniform across large parts of the design or even across the whole project. This means that, for example, the uncertainty associated with bulk material quantities, for one type of material, will be more or less the same across related parts of the estimate. In the same way, the extent to which market rates have been assessed, using a mix of past projects, budgetary quotations, benchmarking, firm quotes or other methods, will be fairly uniform for related classes of materials and equipment.

The level of uncertainty affecting quantities or rates across a large part of a project can be used as a high level representation of cost uncertainty in a risk model. The approach recommended here is to break down the project cost as a whole in such a manner that uncertainty in bulk material quantities, unit rates, productivity assumptions, major equipment prices and other major estimating inputs can be applied to those parts of the cost that they affect at as large a scale as possible. This process starts with the simplest breakdown possible and only introduces more detail where it is necessary to allow distinct uncertainties to be applied.

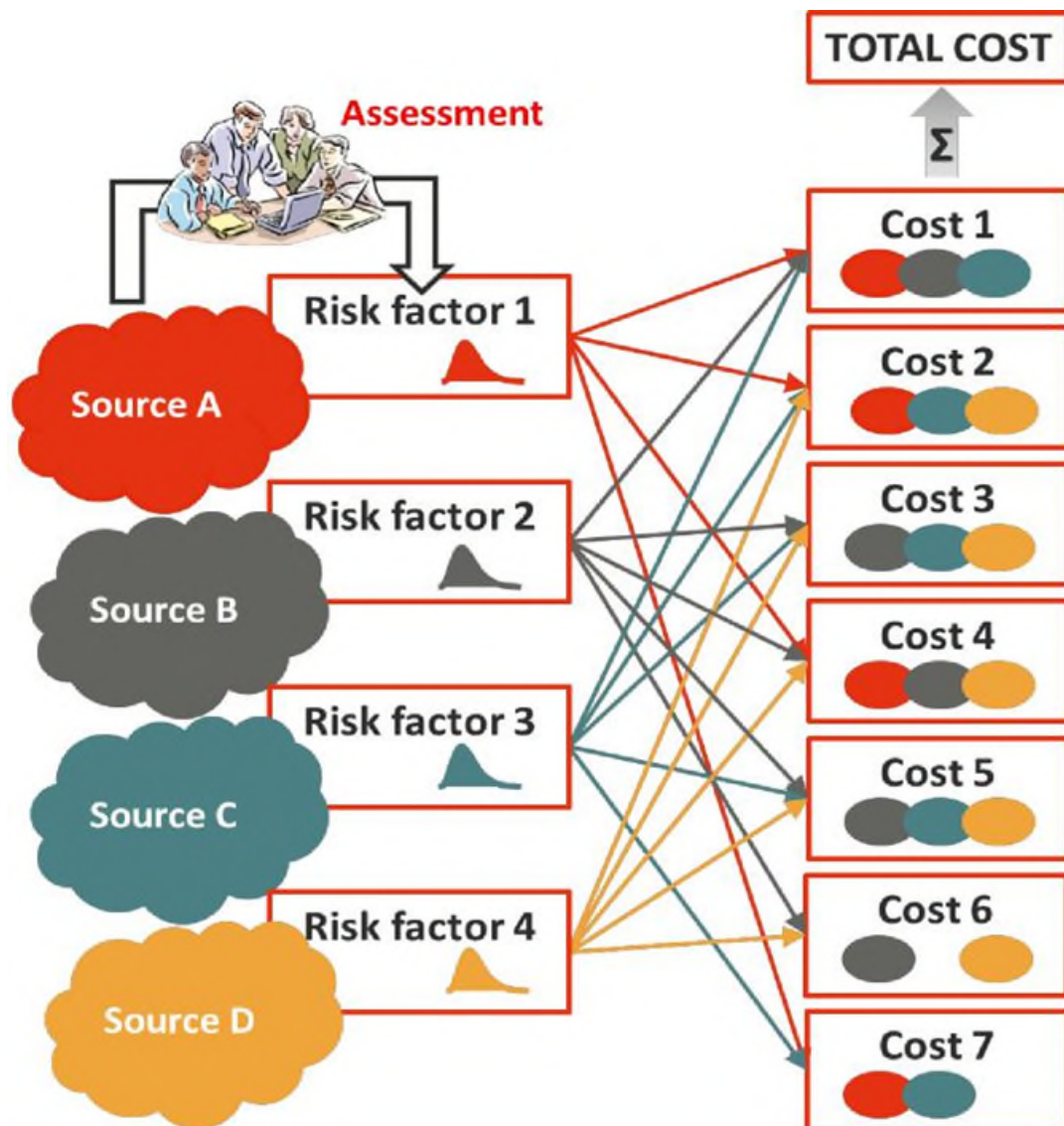
By only breaking the estimate into the minimum number of parts required to fit the way uncertainty is spread across the project, the model remains as simple as it can be while having enough detail to represent the separate areas faithfully. For instance, if the uncertainty about bulk earthworks quantities was higher in one area of a project than in another, perhaps due to having better geotechnical information about one region than another, quantity uncertainty in those two areas might be handled separately and the costs in the two areas would have to be separated out to allow this to be done.

The ultimate in simplicity would be a single assessment of the uncertainty in the total cost. However, this would not provide sufficient detail to represent the important uncertainties affecting a project's cost realistically. Several separate areas of uncertainty would be applied to a single total cost so the uncertainties and the cost, would be out of balance with one another. One would be very coarse - a single item - and the other would be much more granular.

As noted earlier, research into project cost sensitivity has found that very few projects are subject to significant uncertainty in more than about twenty basic estimating inputs. Highly detailed models, with a large number of individual events and cost lines, often include many more variables than there are important sources of uncertainty. In mathematical terms, they are overdetermined and several of their inputs carry

exactly the same information so the related inputs can be replaced with a single factor that represents them all. The principle of a clean model structure is illustrated in **Figure 9** where each distribution in the model is independent of the others and can be applied directly to the costs it affects.

**Figure 8: Clean model structure**

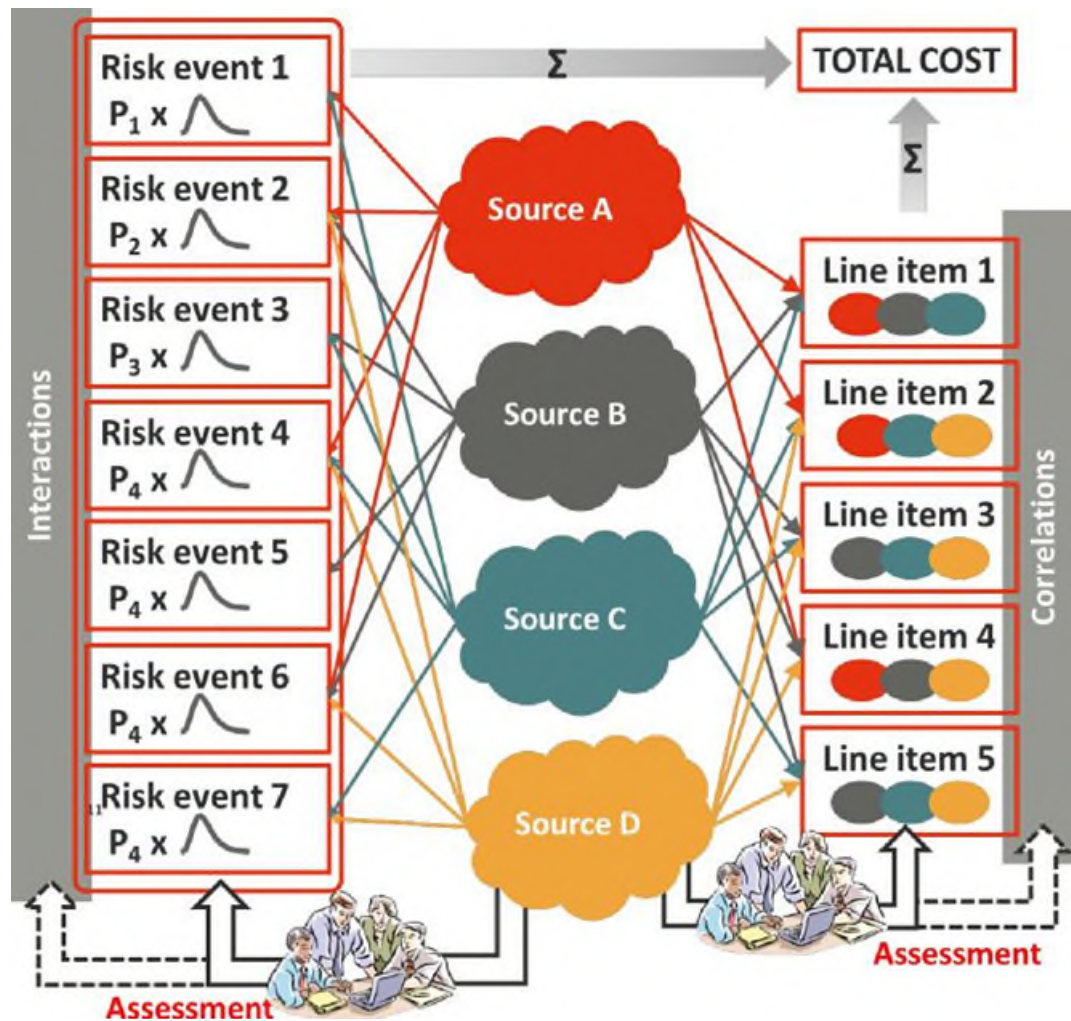


In this structure, uncertainty is assessed in terms of its root causes, which stem from the level of definition achieved in the engineering design and the degree of confidence with which future prices and rates can be established. Those undertaking the assessment will know or should take the trouble to inform themselves of these matters.

A perfect separation of risk factors might not always be possible but something close to it is generally feasible. Even if it is not, having a few underlying correlations, such as market related uncertainties linking prices and rates, is far preferable to the poor structure illustrated in **Figure 10**.



Figure 9: Poor model structure



In this structure, uncertainty is assessed in terms of values that are subject to multiple sources of uncertainty. Manifestations of the one source of risk play a role in multiple risk events and in numerous estimate line item distributions. This gives rise to interactions between the risk events and correlations between the line items, which might be noted but are often overlooked. Even when they are noted, they are very difficult to model realistically.

Unless the interactions and correlations are carefully analysed, a model constructed in this way will always lack important features of the real world. If the interactions and correlations are carefully analysed, they will absorb a large amount of effort and, even then, it might not be possible to represent them faithfully in a model. There is no reliable approach to subjective assessment of partial correlations, which is at best an abstract concept to most people. Without a means to do this, the connections between line items on the right hand side of **Figure 10** cannot be assessed let alone included in a model.

Models of the form illustrated in **Figure 10** effectively represent each big source of uncertainty many times over in a lot of separate potential events or estimate lines. As outlined earlier, it is all but impossible to assess and represent correlations realistically between large numbers of line-items or to allow for the number and complexity of the dependencies that can arise among a large number of risk register entries. To build a model that ignores these is to leave out a large part of the way risk affects a project's costs. To build a model that deals with these interactions and correlations realistically is at best cumbersome, time consuming and often impossible for all practical purposes.



## 6.2. General structure

### Uncertainties

The first step in designing a cost risk model is to determine how best to describe the uncertainties that affect the cost. This is best approached from the top down, using broad descriptions of uncertainty that cover major parts of the cost wherever possible and introducing detail where, but only where, it makes a significant difference to the quality and realism of the model. Matters that attract a lot of attention are not always the most significant.

As noted earlier, infrastructure cost estimates are almost always subject to uncertainty in:

- Quantities of materials bought, moved and placed.
- Unit rates for the purchase of materials, equipment or labour.
- Productivity rates for labour and plant.
- Running rates for management, overheads and temporary facilities.
- Lump sum costs of major purchases.
- The duration of the work.

There is no single way to arrive at a sound cost risk model but, as a general guide, it is useful to start with these factors and consider whether, for a particular project:

- They are all relevant.
- Which bulk quantities are important (are subject to appreciable uncertainty and affect a significant part of the cost).
- Which rates are important (are subject to appreciable uncertainty and affect a significant part of the cost).
- Whether it is sufficient to represent the level of uncertainty in these quantities and rates across the whole job or it is necessary to split the project into physical parts, phases or types of work (green and brown field for instance) for some uncertainties.
- Which lump sum amounts, such as major equipment purchases, it will be useful to split out.
- Whether time dependent costs can be analysed as a whole across the entire duration or will be subject to markedly different levels of uncertainty at different stages in the work.

Subcontract items can create some confusion. Once a subcontract is signed, some risk will usually be transferred to the subcontractor. However, subcontracts might not be in place at the time an estimate is prepared as a project is seeking approval. In this situation, uncertainty in quantities and rates will play a role in estimating the cost of the subcontract. If it assumed that the subcontractor will take account of the same factors as the project, the value of the subcontract will be subject to the same quantity uncertainty as if the work was being performed by direct labour as well as often being subject to uncertainty about subcontractors' mark ups and overheads.

It is important to remember that the ranges represented by risk factors affecting rates and prices are intended to represent uncertainty due to state of the design and procurement information available at the time an estimate is prepared. They represent the potential variation that could arise between what is in the estimate and the actual cost if orders were to be placed and work was to be carried out immediately. There is a danger that people assessing ranges on rates and prices will fall into the trap of including in their assessment variations that will arise due to escalation. Escalation might also be subject to uncertainty but it is quite distinct from uncertainty associated with material specifications, commercial arrangements with suppliers and similar matters.

**Guidance Note 4 – Escalation** discusses these matters.

### 6.3. Linking cost drivers to risk drivers

Risk is the effect of uncertainty on objectives. Risks should not be confused with things that are simply expensive. For example, while the pavement in many road projects may make up the largest proportion of the cost, it will only increase and drive up the total cost significantly if the scope grows or the design changes appreciably. This will generally not happen if development and project planning are done well. If the scope is well-defined, subgrade conditions are well known, material of known quality is known to be readily available, etc., then the range of possible cost outcomes will be small in percentage terms. It is the level of scope definition and geotechnical investigation that is the source of the uncertainty, not the fact that the pavement is a large proportion of the total cost.

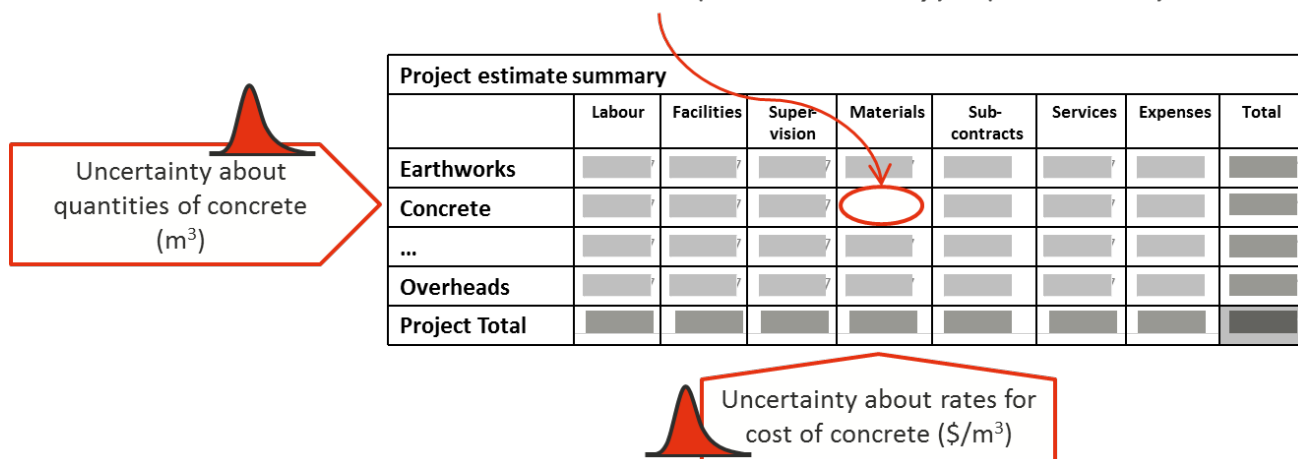
One of the core principles of a risk factor approach is to provide a clean relationship between the factors that represent uncertainties, such as those listed earlier, and the costs that they affect. There are several ways this can be accomplished. The example models described in this section show one way to do this. Estimators and analysts will use their ingenuity to develop methods that suit them and their other systems while continuing to pay attention to the principles of simplicity and a clean structure.

A useful approach that fits many projects is to use a cost matrix that splits costs one way to match the description of quantity uncertainties along the other axis to allow the application of rate uncertainties. An example of this is illustrated in **Figure 11**.

**Figure 10: Risk model structure**

**Cost estimating relationships (e.g. Cost = Quantity x Unit rate)**

$$\text{Simulated cost} = \text{Base estimate} \times (1 + \Delta\text{Quantity}) \times (1 + \Delta\text{Rate})$$



Project estimate summary								
	Labour	Facilities	Super- vision	Materials	Sub- contracts	Services	Expenses	Total
Earthworks								
Concrete								
...								
Overheads								
Project Total								

In this structure, labour rate and productivity uncertainty are applied to the labour column. Productivity uncertainty is also applied to any other columns related to direct labour. Other rate uncertainties are applied as appropriate to other columns. Quantity uncertainties are applied across the rows, combining with the rate factors applied in each column. This means that, as shown for one cell in **Figure 11**, the combined effect of quantity and rate uncertainty is applied to each bulk material cost. In the same fashion, uncertainty in quantity, productivity and the labour rate are applied to the labour cost while productivity and plant rate uncertainties are applied to plant costs and so on.

Lump sum costs, such as major equipment purchases, are readily incorporated into this structure. All that is required is to use formulae in the relevant cells to apply lump sum uncertainty to the cost it affects, generally expressed as a percentage variation. Major equipment items, such as tunnel boring machines or large power transformers, are often assessed separately as the commercial arrangements with their suppliers and the nature of the market for each one may be quite different to that of the others.

To build a model on this basis all that is required is to summarise the estimate in a form that

separates groups of costs that are subject to separate quantity and rate uncertainties. Some of the common ways of splitting costs up were listed in **Table 1** at [Section 4.2](#).

It is also feasible to take a line item estimate summary and apply the risk factors simulated in one part of the model to each line they affect. This has the same effect as it applies the same distributions to all lines with common underlying sources of uncertainty. They are all driven by a single value of each risk factor, a single distribution in a model. While the number of risk factors is not increased by such an approach, it does result in a rather bulky model as each factor has to be written into a formula for each line to which it applies. It is obviously preferable to try to create a summary such as that in **Figure 11**, which summarises costs as far as they can be while preserving the structure required to allow the risk factors to be applied cleanly.

## Lump sums

Lump sum costs might represent the purchase of major items of installed equipment, specialist construction equipment or the purchase of land and buildings. It is rarely necessary to break these down further and uncertainty in the cost of individual items or classes of items can usually just be assessed directly.

If freight costs are significant, perhaps for a remote site or equipment requiring specialised transport, unloading and placement, or if freight costs are subject to different commercial conditions to the cost of the items themselves, it may be worthwhile breaking out freight costs and assessing their uncertainty separately.

## Risk events for inclusion

If a project is subject to potential events that can have an appreciable effect on the project cost, they can easily be included in the model. It is important to consider first whether:

- They have a high enough likelihood to warrant including in the contingency as opposed to being rare background risks that would be considered ‘business as usual’ operating conditions, such as a risk of epidemic disease.
- They are not so large and likely to occur that they should be treated as separate contingency items outside the main project account, as outlined in [Section 4.4](#), such as a requirement to reduce road noise levels to a much lower level due to a new standard being imposed for political reasons.

Care may be required to avoid the unnecessary use of events in risk models simply by default, because risk registers have often been used as the basis of risk models in the past. It is difficult to be prescriptive but, if an event risk cannot affect a project’s cost by more than a few percent, it will often fall naturally into one of the risk factor range assessments as one of a number of discrete uncertainties felt by the project as an aggregate uncertainty in the quantities or rates affecting a substantial part of the cost as a whole.

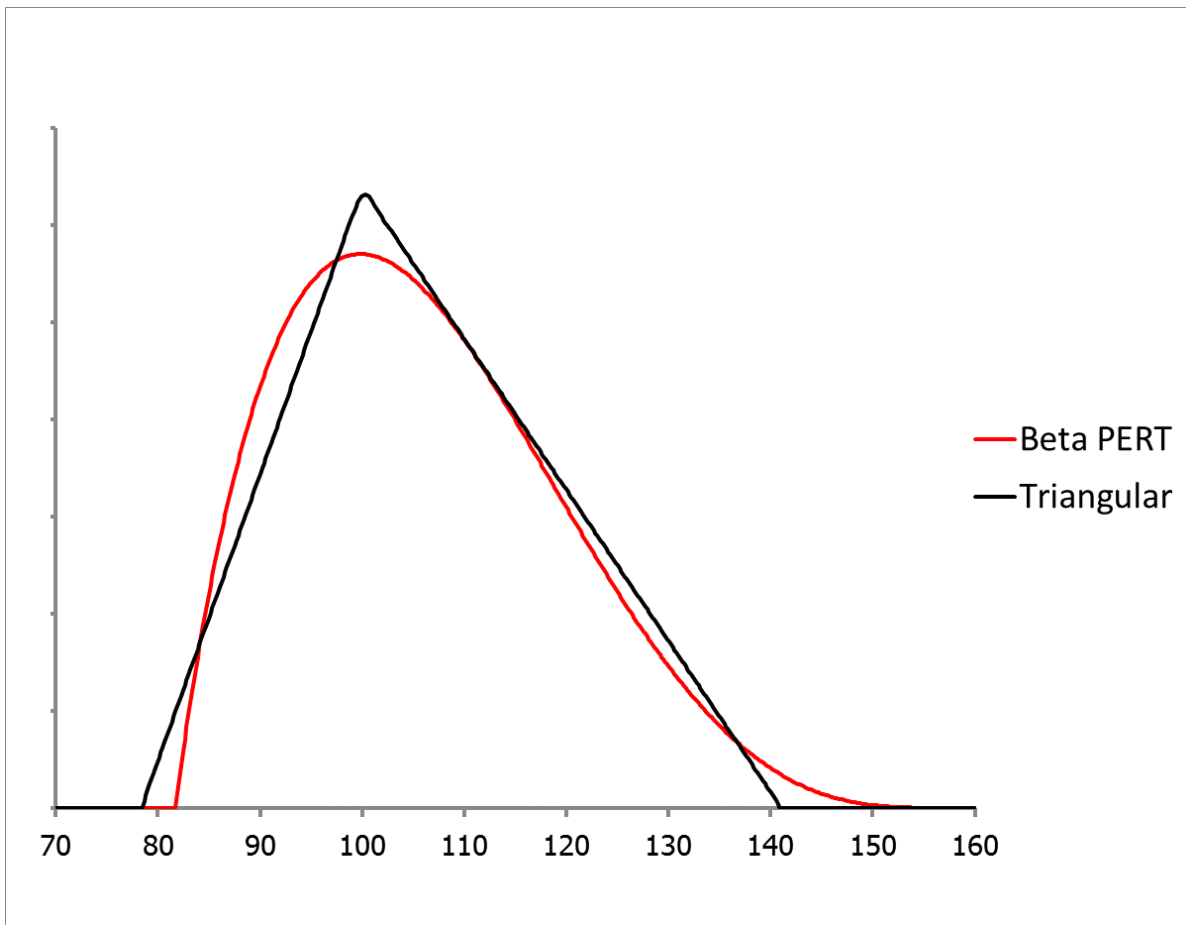
It will often be useful to analyse such events separately for the purposes of day to day management or to understand the costs and benefits of strategies for treating the risks, supporting operational management decision making. This is a separate exercise to the task of assessing the overall contingency required for a project as a whole. There is no need to clutter the project contingency analysis with the details of day to day management decision making. Important as they are in their own right, not all such decisions are significant enough to warrant separate attention when assessing project total funding requirements.

## 6.4. Choice of uncertainty distribution

A probability distribution represents the likelihood of an uncertain quantity taking on values within the range that can arise. There is no objective basis for choosing one particular distribution shape to represent an assessment of the uncertainty in part of a project’s cost. Two shapes in common use for routine cost modelling are the Beta PERT and triangular distributions. These are illustrated in **Figure 12**, which shows

examples of the two distributions with the same P10, Mode and P90 values, so they represent the same inputs.

**Figure 11: Distribution shapes**

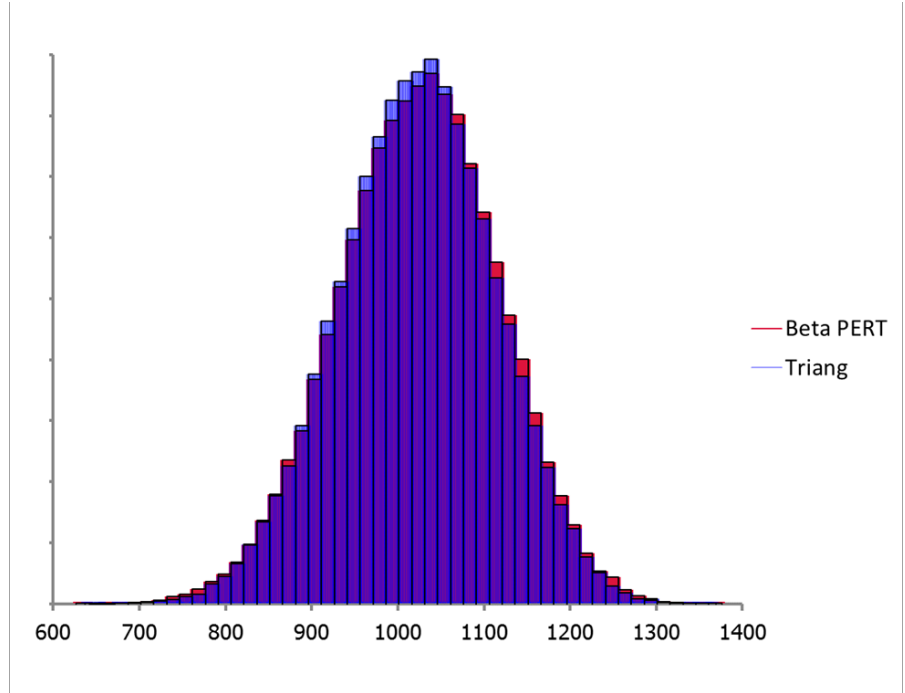


The differences between the two shapes are minor.

A simple simulation, which combines ten distributions based on the inputs listed in **Table 11**, using two distribution shapes shows that the difference the distribution shape makes to a model output is usually negligible. The output of a model that uses these three point estimates to specify, in one case, triangular distributions and, in the other, and Beta PERT distributions is shown in **Figure 13**.

**Table 11: Data for illustration of effect of distribution shape**

P10	Mode	P90
20	100	140
30	120	140
0	30	90
10	50	50
80	140	150
150	200	240
160	180	210
20	25	25
145	175	205
50	70	110



**Figure 12: Effect of distribution shape on model output**

In the model used to generate the graphs in **Figure 13**, ten triangular distributions defined by the P10, Mode and P90 values in **Table 11** were added up and ten Pert distributions defined by the same P10, Mode and P90 values were also added up. The two sets of distributions and their sums were evaluated using Monte Carlo simulation in @Risk. It is clear that the data in **Table 11** generated essentially the same distribution for the total cost using triangular distributions as it did using PERT distributions. The choice of shape for the individual distribution had no material effect on the output. This is almost always the case, except where the extreme values or tails of the output distribution are important.

Project funding decisions are generally not concerned with outcomes in the extreme tails of the distribution where choice of distribution shape may make a difference, but focus their attention between the 10th and 90th percentiles<sup>28</sup>. The choice of distribution will rarely make a difference to the outcomes within this range.

There is a useful variation of the triangular distribution called Trigen in @Risk. The Trigen distribution requires five parameters:

- A low value
- The most likely
- A high value
- The probability that the parameter value could be below the low value.
- The probability that the parameter value could be above the high value.

Because the analyst can discuss what probabilities the experts would use to define optimistic and pessimistic values, the Trigen adapts to each expert's concept of these points. The most common optimistic and pessimistic points used to elicit range assessments and implement distributions in a model are the P10 and P90.

<sup>28</sup> Cooper D, Bosnich P, Grey S, Purdy G, Raymond G, Walker P, Wood M, 2014 Project Risk Management Guidelines: Managing Risk with ISO 31000 and IEC 62198 2nd Edition

In the context of determining the most appropriate distribution for a given risk factor, all of the parameters required to define a Trigen distribution will have been identified within the risk workshop and recorded within **Table 9**. The Department considers that, unless it is known that a distribution conforms to a certain shape, the Trigen, Triangle or PERT are all a sensible choice of distribution to allocate to a given risk factor.

**Appendix D** provides a further discussion of different types of distributions, and their applications.

## 6.5. Account for correlation between WBS element costs to properly capture cost risk

As previously discussed, valid Monte Carlo simulation requires that the dependencies between the model inputs be understood and included in the model. One of the main reasons for the development of risk factor models is that they avoid the need to incorporate complicated correlations into risk models. Risk driver/risk factor methods enable this because true root causes are, as far as possible, chosen to be statistically independent and so that they can be combined using simple functional relationships.

The most common use for correlations in risk factor models is to link market rate uncertainties for bulk materials, plant and possibly subcontractors' distributables and overheads. If the supply of quarry materials, concrete and steel in a region are all driven by the level of similar project activity that will be underway when the work commences, each one can be expected to take up a similar position within its own range. They will probably all be at the high end, all at the low end or all at a mid-point of their expected ranges. This can be implemented using correlations in the models, as is shown in the example models.

In some circumstances, labour rate uncertainty will also be correlated with uncertainty in material supply and plant rates. This is handled in the same way.

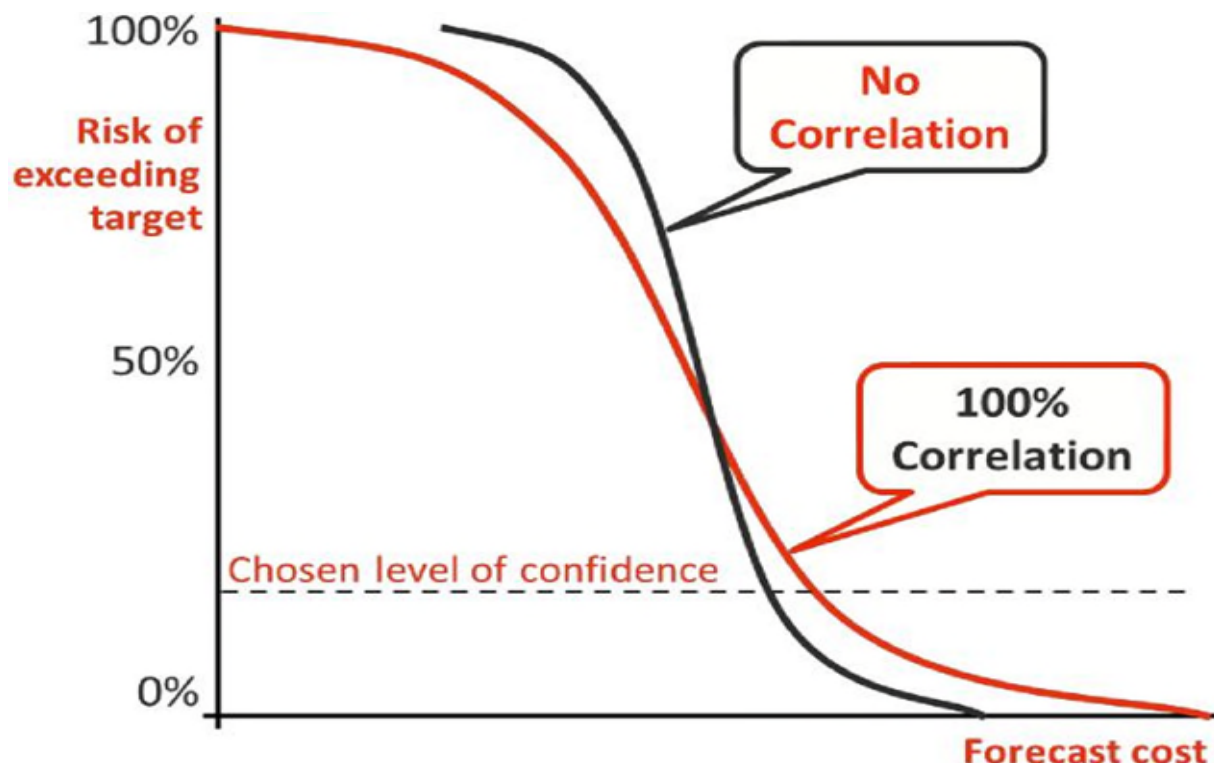
It sometimes happens that uncertainty in the productivity of one class of labour is different to the uncertainty in the productivity of another class of labour, perhaps green field and brown field or across separate disciplines. While the level of uncertainty in the productivity of each class of labour may be different, if they all depend on site conditions, such as congestion or the quality of planning and co-ordination, they might all tend to be high, all be low, or all fall towards the middle of their separate ranges in unison. Here too, it is not uncommon to use correlation to represent this linkage in a model.

This simple recognition of a single common source of uncertainty affecting a set of related risk factors is quite different to the challenge of understanding correlations between work package costs that depend on uncertainty in quantities and rates for two or three separate commodity unit rates such as concrete and steel reinforcing. The correlation between parts of the structure that have little or no reinforcing and parts that are heavily reinforced will be affected by the fact that steel plays a large role in one of them and a minor role in the other and the cost of concrete dominates the cost of one area and is a lesser proportion of the cost in the other. In addition, uncertainty in the steel component will be correlated to some extent with the cost of steel structures. In these circumstances, modelling the concrete cost uncertainty separately from the steel cost uncertainty is simpler and cleaner than modelling the work package cost uncertainties and trying to assess a realistic level of correlation between them.

The point has been made earlier that assessing partial correlations realistically is practically impossible. When the connection between separate uncertainties is quite strong, as with unit rates for bulk materials in a market dominated by activity of the same sort as the project being analysed, it is common to implement correlations with coefficients of 100%, i.e. complete correlation.

The sensitivity of an analysis to correlation can be tested by setting it to zero and then to 100% and comparing outputs, as illustrated in **Figure 14**.

Figure 13: Testing the significance of correlation



This shows the cumulative distribution of forecast cost outcomes from a model with correlation between some of the factors set to 100% and then set to zero. The gap between the two curves at the level of confidence chosen to fix the contingency shows how significant an effect correlation has on the outcome. If the gap is negligible, correlation can be ignored. If it is substantial and 100% correlation is considered meaningful then the correlated version can be used.

If it is felt that correlation exists but it is less than 100%, a contingency in between the amounts represented by the two curves can be chosen by decision makers. This is a subjective judgement

as is the application of partial correlations. A sensitivity analysis of this sort provides more context for the exercise than the subjective assessment of correlations in isolation. To most people, correlations are poorly understood abstract concepts. The sensitivity approach condenses the exercise into a single assessment.

Readers should refer to **Appendix C** which provides a more detailed discussion on the topic of correlation.

## 6.6. Setting the seed in the model

Modelling tools such as @Risk produce numbers from one iteration to the next that appear random even though they are reproducible if the sequence starts from the same initial value or seed. They are more strictly referred to as pseudorandom rather than random. The default setting for @Risk is to randomly choose a new seed for each simulation and under most circumstances there is no need to specifically set the seed. The seed governs the sequence of random numbers produced by the random number generator. By fixing the seed, exactly the same sequence of random numbers can be repeated each time the simulation runs.

Setting a fixed seed value is useful for scenario/sensitivity testing where it is important to control the simulation sampling environment. For example, an analyst may wish to simulate the same model twice, changing only one distribution function. By setting a fixed seed, the same values will be sampled from all distribution functions during each iteration, except the one that was changed. Therefore, any differences in



the results between the two runs will only be due to the changed argument values of the single distribution function and will not be affected by the inherent variability of the modelling process itself<sup>29</sup>.

## 6.7. Example risk factor model

A risk factor model aims to provide a clean relationship between the factors that represent uncertainties, and the cost that they affect. Four example models accompany this guidance note summarised in **Table 12**.

**Table 12: Example model summary**

Model	Risk factor	Description
1	Risk factor model 1.xlsx	A road project in which quantity uncertainty is assessed for each area of the project and combined with rates uncertainty. Uncertainty in duration is combined with uncertainty in the running rate of overheads and one event risk is included. This model also demonstrates contingency allocation between cost lines, which is discussed in appendix G.
2	Risk factor model 2.xlsx	Using the same base estimate as the model number 1, this model uses quantity uncertainty assessed for separate disciplines.
3	Risk factor model 3.xlsx	This model is the same as model number 2 but, instead of a direct assessment of duration uncertainty, uses a duration distribution based on the output of a separate schedule risk analysis modelled in Primavera Risk Analysis, once known as Pertmaster, and imported into Excel.
4	Risk factor model 4.xlsx	A model of a rail project based on quantity and rate uncertainties that uses an imported schedule distribution in the same way as model number 3.

The first three models have separate tabs for:

- The base estimate.
- A summary of the estimate designed to match the risk factors being used, so the factors can be applied cleanly to the costs they affect.
- A table with the same structure as the summary estimate in which the risk factors applicable to each cost entry in the summary are combined.
- A model table that also has the same structure as the summary estimate and combines the base estimate values with the risk factors from the previous tab.
- A risk factors tab in which the three-point estimate inputs to the model are used to define risk factor distributions and the factors are given Excel range names.
- A tab containing a chart and functions that generate an output distribution and table of percentiles for the total cost.

<sup>29</sup> Palisade (2015), @Risk User's Guide Version 7 Palisade Corporation



The first model includes a tab showing how the overall contingency can be associated with work package subtotals.

The third model includes a schedule distribution tab in which the output of a Primavera Risk Analysis (PRA) model of schedule uncertainty has been used to recreate the schedule distribution in @Risk.

The fourth model has the risk factors written directly into the model sheet illustrating just one alternative way that costs and risk factors can be combined.

The risk factors are all modelled using a trigon distribution based on assessments of the P10, Mode and P90 values of the quantities they represent, as discussed in [Section 5.4](#). In the equation below (**Figure 15**), the Quantity and Rate variations for a cost that is subject to uncertainty in the bulk material quantity and in the unit rate for the material are converted to multiplicative factors by adding 100% to the percentage variation sampled from each input distribution.

**Figure 15: Risk factor application**

$$\text{Simulated cost} = \text{Base estimate} \times (1 + \Delta\text{Quantity}) \times (1 + \Delta\text{Rate})$$

The risk factors tabs also include correlation matrices linking rate uncertainty across several inputs and, in the first three models, the discrete risk event included in the model. The risk event has a defined probability of occurring and an uncertain impact value if it does occur.

The fourth model does not use correlations although it could if this was deemed appropriate. It has a separate tab for model parameters where the start and end dates of the two parts of the job are entered and used in the two schedule models, the longest one of which drives the overheads. The proportions of overheads that are subject to schedule variation are also entered here as parameters and used in the model to modify the schedule variation's impact on time related costs. This avoids the need to separate out the time dependent and other costs from one another in the overheads.

The schedule tabs in models number 3 and number 4 use the @Risk function RiskCumul() to recreate from the PRA histogram of completion dates a distribution of the duration of the project. This is turned into a distribution of the simulated duration relative to the base duration using the planned start and end dates. That relative variation, the duration risk factor, is used to drive the overheads, management costs and temporary facilities costs.

The summary output tabs, that create the cumulative distribution and percentile table, and the schedule tabs that recreate the PRA output, have notes embedded explaining key points about linking them into the model and importing the PRA data.

## 6.8. Step through of model

The model accompanying this guidance note has been created using @Risk<sup>30</sup>. As such, naming conventions to create functions and distributions reflect that particular piece of software. The model could easily be replicated in other software noting that some add-ins such as ModelRisk will allow @Risk files to be imported directly with naming conventions for formulas automatically changed to suit the ModelRisk software.

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<sup>30</sup> The examples in this guidance note have been developed using the proprietary software programme @Risk and are used for demonstration purposes only. The Department does not endorse @Risk and acknowledges the availability of similar software tools.

The structure of the model itself is the key to the risk factor approach and will remain the same whichever software simulation tool is used.

The following section demonstrates how a risk factor approach can be applied to an estimate. Risk factor model 2.xlsx has been used as the basis of this explanation. It is suggested that readers open the spreadsheet model while reading through this explanation.

Note that the process has been described as a series of steps. In reality the process will be both non-sequential and iterative. For example, as an estimator begins to build an estimate, an idea of the risk factors likely to affect the project will begin to emerge. This in turn will affect how the estimate is best structured and summarised in order to provide a clean relationship between the factors that represent uncertainties and the costs that they affect.

### Step 1 – Base estimate

A first principles estimate is the starting point before a risk analysis is undertaken. This particular model builds up costs for each element using the following basic inputs:

- Staff
- Labour
- Plant
- Bulk Materials
  - Concrete
  - Steel
  - Other
- Subcontract
- Other

An extract of the base estimate is shown in **Figure 16:**

**Figure 14: Base estimate extract – risk factor model 2.xlsx**

		Bulk materials								
Sunk costs		Staff	Labour	Plant	Concrete	Steel	Other	Subcon	Other	TOTAL
									6,374,775	6,374,775
A0000	Initial design	-	-	-	-	-	-	-	1,568,726	1,568,726
A0000	Detailed design (70%)	-	-	-	-	-	-	-	2,126,023	2,126,023
A0000	Consulting fees	-	-	-	-	-	-	-	101,708	101,708
A0000	Environmental and community consultations	-	-	-	-	-	-	-	2,578,318	2,578,318
	Provisions	-	153,863	146,669	1,808,144	-	-	-	820,776	2,929,452
P1010	Onsite rework of reinforcing	-	143,834	39,785	-	-	-	-	-	183,619
P1020	Corrosion protection	-	-	-	1,808,144	-	-	-	-	1,808,144
P1030	Inclement weather	-	-	-	-	-	-	-	372,100	372,100
P1040	Delay costs	-	10,029	106,884	-	-	-	-	-	116,913
P1050	Bank guarantees	-	-	-	-	-	-	-	448,676	448,676
	Establishment	6,122,832	139,526	745,513	443	-	457,686	669,281	1,083,692	9,218,975
	Cable relocation	-	-	-	-	-	-	320,659	-	320,659
J1020	Cable and connections	-	-	-	-	-	-	211,451	-	211,451
J1030	Remove existing cabling	-	-	-	-	-	-	109,208	-	109,208
	Site establishment	8,829	83,777	69,473	443	-	137,686	263,806	-	564,014
A2400	Mobilise site office 1	-	2,647	1,567	244	-	35,985	37,921	-	78,364
A2400	Mobilise site office 2	-	23,106	22,542	199	-	95,961	70,220	-	212,028
A2300	Site offices	-	-	3,892	-	-	-	-	-	3,892
A2300	Site offices 1	-	-	8,752	-	-	-	-	-	8,752
A2300	Site offices 2	-	-	2,547	-	-	-	-	-	2,547
A2100	Site supervisor	8,829	-	-	-	-	-	-	-	8,829
E2001	Safety zones	-	5,761	733	-	-	2,214	-	-	8,708
E2002	Set up environmental controls	-	5,626	655	-	-	1,438	-	-	7,719
E6010	Soil testing	-	1,285	294	-	-	-	155,665	-	157,244
E6020	Initial earthworks	-	45,352	28,491	-	-	2,088	-	-	75,931

### Step 2 – Determine risk factors

As the estimate is built up from first principles, many of the major sources of uncertainty, such as productivity rates, will become obvious because considering them is part of normal estimating practice. For this particular example project, 20 risk factors have been identified as shown in **Figure 17.**

**Figure 15: Hypothetical risk factors – risk factor model 2.xlsx**

	A	B	C	D	E	F	G
1		<b>Variation</b>					
2	<b>Description</b>	<b>Optimistic</b>	<b>Likely</b>	<b>Pessimistic</b>	<b>Simulated</b>	<b>Factor</b>	<b>Excel name</b>
3	Provisions	0%	0%	20%	9%	109%	Provisions
4	Preliminaries rate	-5%	0%	15%	4%	104%	Prelim_running
5	Duration	-10%	0%	25%	6%	106%	Duration
6	Engineering scale	-10%	0%	20%	4%	104%	Q_Eng
7	Plant and site equipment quantity	0%	0%	25%	11%	111%	Q_Plant
8	Earthworks quantity	-5%	0%	15%	4%	104%	Q_Earth
9	Concrete quantity	-5%	0%	10%	2%	102%	Q_Conc
10	Retaining walls quantity	-5%	0%	20%	6%	106%	Q_Walls
11	Pavement quantity	-5%	0%	5%	0%	100%	Q_Pave
12	Barriers quantity	0%	0%	10%	4%	104%	Q_Barrier
13	Piling quantity	-5%	0%	15%	4%	104%	Q_Pile
14	Structural steel quantity	-5%	0%	10%	2%	102%	Q_Steel
15	Concrete rate	-10%	0%	15%	2%	102%	R_Conc
16	Steel rate	-10%	0%	10%	0%	100%	R_Steel
17	General bulks rate	-5%	0%	10%	2%	102%	R_General
18	Plant rate	-10%	0%	5%	-2%	98%	R_Plant
19	Subcontract rates	-10%	0%	5%	-2%	98%	R_Subcon
20	Staff rate	0%	0%	5%	2%	102%	R_Staff
21	Labour rate	0%	0%	10%	4%	104%	R_Labour
22	Productivity	-5%	0%	25%	9%	109%	Productivity
23							
24							
25	<b>Major risks</b>						
26	Noise walls required at southern end			Net impact	\$ .0M		
27	Impact	\$ .5M	\$ 1.2M	\$ 2.0M	\$ 1.2M		
28	Occurrence	Prob =	15%		0		

The uncertainty in each risk factor is represented by a probability distribution, in this case modelled as a Trigen distribution. The value in the “Simulated” column for the risk factor “Provisions” is thus:

=RiskTrigen(B3,C3,D3,10,90,RiskName(A3))

This value will change with each iteration of the simulation. The value in the “Factor” column is the number one plus the value in the simulated column. It will also vary with each iteration of the simulation.

One major risk event, the requirement for noise walls, which cannot be captured within the risk factors, has also been identified and included.

It has been judged that the rates for concrete, steel, general bulks, plant, and sub contract are correlated with each other and this has been captured within a correlation matrix as shown in **Figure 18** below.

**Figure 16: Correlation matrix – risk factor model 2.xlsx**

@RISK Correlations	Concrete rate in \$E\$15	Steel rate in \$E\$16	General bulks rate in \$E\$17	Plant rate in \$E\$18	Subcontract rates in \$E\$19
Concrete rate in \$E\$15	1				
Steel rate in \$E\$16	1	1			
General bulks rate in \$E\$17	1	1	1		
Plant rate in \$E\$18	1	1	1	1	
Subcontract rates in \$E\$19	1	1	1	1	1

### Step 3 – Summarise estimate

The aim here is to summarise the estimate in a logical manner that allows the major sources of uncertainty to be linked to the costs they affect. This example estimate has been summarised under the following headings:

- Establishment and preliminaries
- Area 1
- Paths
- Viaduct
- Area 2 (Bridge)
- Superstructure

This structure works particularly well for this example. The summary headings are separate areas and can almost be seen as discrete ‘sub-projects’ in themselves which makes them easy to visualise, making it simpler to consider the impact of uncertainty on the cost inputs. Other projects may lend themselves to a different summary structure. The key points when summarising are:

- Break, or aggregate into components that are reasonably homogenous and, where possible, independent of one another.
- Identify how it will be best to apply the risk factors, for example by summarising the estimate by areas, parts, phases, types of work, or some other logical structure that reflects the type of risk exposure.
- The aggregation should not just be a roll-up of the project estimate into a standard breakdown; certain types of cost may warrant more detail than others.

The summary base estimate for this model is shown in **Figure 19**.

**Figure 17: Summary base estimate – risk factor model 2.xlsx**

	Staff	Labour	Plant	Bulk materials			Subcon	Other	TOTAL
Sunk costs	-	-	-	-	-	-	-	-	-
Provisions	-	153,863	146,669	1,808,144	-	-	-	6,374,775	6,374,775
Establishment and preliminaries	-	-	-	-	-	-	-	820,776	2,929,452
Preliminaries time independent	8,829	83,777	69,473	443	-	137,686	584,465	-	884,673
Preliminaries time related	6,114,003	55,751	676,040	-	-	320,000	84,816	1,083,692	8,334,302
<b>Direct costs</b>									
<b>Area 1</b>									
Design	2,103,961	-	-	-	-	-	-	-	2,103,961
Special plant	-	2,277,480	1,719,349	-	-	499,255	-	-	4,496,084
Site equipment	-	868,124	472,976	-	-	165,435	10,522	449,451	1,966,508
Environmental management	231,777	30,299	-	-	-	62,559	-	-	324,635
Stormwater drainage	-	2,418	1,341	12,523	-	885	108,691	-	125,858
Earthworks	-	191,007	143,782	-	-	-	799,445	-	1,134,234
Shotcreting	-	-	-	-	-	-	919,039	-	919,039
Retaining walls	-	128,223	1,079	52,090	-	277,397	269,276	-	728,065
Pavement	-	17,537	14,298	-	-	38,897	341,038	-	411,770
Fencing	-	-	-	-	-	-	84,201	-	84,201
Barriers	-	4,459	2,476	-	-	-	83,991	-	90,926
<b>Paths</b>									
Shared path North	-	9,721	516	740	-	22,660	6,257	-	39,894
Shared path South	-	2,975	307	234	-	6,589	1,940	-	12,045
Pedestrian areas North	-	5,055	-	-	-	38,281	-	-	43,336
Pedestrian areas South 1	-	8,649	-	-	-	87,957	-	-	96,606
Pedestrian areas South 2	-	1,947	-	-	-	15,322	-	-	17,269
Boardwalk and lookout	-	41,521	-	-	67,272	102,583	72,950	-	284,326
Signs and linemarking	-	-	-	-	-	-	37,825	-	37,825
Landscaping	-	6,552	2,861	-	-	24,028	25,370	-	58,811
Access track and surrounds	-	1,586,485	1,077,686	6,322	-	1,132,809	47,746	-	3,851,048
<b>Viaduct</b>									
Piling	-	4,793	1,904	258,431	217,997	-	622,412	-	1,105,537
Abutments	-	109,124	10,538	46,263	33,602	16,237	-	-	215,764
Pile caps	-	644,857	136,442	278,563	864,731	72,305	21,085	-	2,017,983
Piers	-	2,783,877	171,342	1,277,729	2,131,777	1,534,420	1,838,712	-	9,737,857
Parapets and crash rails	-	180,255	-	83,889	99,669	1,011,778	-	-	1,375,591
Expansion joint	-	30,912	351	-	-	142,830	-	-	174,093
Connecting slab	-	17,460	252	10,759	12,314	2,360	-	-	43,145
<b>Area 2</b>									
Bridge	-	112,668	83,029	-	-	-	7,060	-	202,757
Piling	-	4,287	1,870	83,241	64,606	-	176,385	-	330,389
Substructure	-	102,296	1,919	47,365	34,961	15,221	-	-	201,762
Pier columns	-	337,687	91,826	323,009	396,991	56,167	22,074	-	1,227,754
Concrete	-	713,708	164,135	507,800	920,432	406,856	645,215	1,489	3,359,635
Barriers	-	16,893	-	-	-	146,079	-	-	162,972
Pedestrian barriers	-	8,419	-	-	-	72,247	-	-	80,666
Expansion joint	-	17,817	-	-	-	88,421	-	-	106,238
Connecting slab	-	19,155	261	9,326	18,467	2,320	-	-	49,529
Abutments	-	100,456	2,165	40,032	31,922	14,184	-	-	188,759
<b>Superstructure</b>									
Girders	-	1,740	683	-	-	131,523	-	-	133,946
Bearings	-	1,480	-	-	-	31,381	-	-	32,861
Deck fit out	-	43,298	2,610	9,117	24,201	18,689	-	-	97,915
Traffic barriers	-	2,359	-	-	-	23,608	-	-	25,967
Pedestrian barriers	-	1,163	-	-	-	9,891	-	-	11,054
Expansion joint	-	2,019	-	-	-	8,844	-	-	10,863
Connecting slab 1	-	28,120	576	21,815	33,551	2,828	-	-	86,890
Connecting slab 2	-	32,722	3,701	24,040	21,241	12,402	-	-	94,106
Total	8,458,570	10,793,408	5,002,457	4,901,875	4,973,734	6,752,934	6,810,515	8,730,183	56,423,676

#### Step 4 – Allocate risk factors

The risk factors are now allocated to the respective inputs that they affect.

Taking as an example the cost item “Shared Path North” (Cell B22 of the Estimate Summary), its total cost is a combination of Labour, Plant, Concrete, Other Bulk Materials, and Subcontract.

For Shared Path North, it has been determined that the uncertainty associated with Labour as a whole is driven by the following risk factors: Earthworks Quantity, Labour Rate, and Productivity. The distributions for those three risk factors are multiplied together to give the total uncertainty for the labour component of “Shared Path North” (=Q\_Earth\*R\_Labour\*Productivity).

Similarly, the uncertainty associated with Plant is driven by Earthworks Quantity, Plant Rate, and Productivity. Multiplying these together gives the uncertainty for the plant component of “Shared Path North” (=Q\_Earth\*R\_Plant\*Productivity). The process is repeated for each component of the cost.

**Figure 20** shows the allocation of risk factors to their associated costs (refer to the tab “Risk factor allocation”). A number of cells are shaded, the shaded cells have no cost in the estimate summary so no risk factors need be applied to them.

**Figure 18: Allocation of risk factors – risk factor model 2.xlsx**

	Staff	Labour	Plant	Concrete	Bulk materials		Subcon	Other
					Steel	Other		
Sunk costs								100%
Provisions		109%	109%	109%				109%
Establishment and preliminaries								
Preliminaries time independent	104%	104%	104%	104%		104%	104%	
Preliminaries time related	111%	111%	111%			111%	111%	111%
Direct costs								
Area 1								
Design	107%							
Special plant		126%	118%			114%		
Site equipment		126%	118%			114%	109%	122%
Environmental management	107%	118%				107%		
Stormwater drainage		118%	111%	104%		107%	102%	
Earthworks		118%	111%				102%	
Shotcreting		118%	111%				102%	
Retaining walls		121%	113%	109%		109%	104%	
Pavement		114%	106%			102%	98%	
Fencing		119%	111%				102%	
Barriers		119%	111%				102%	
Paths								
Shared path North		118%	111%	107%		107%	102%	
Shared path South		118%	111%	107%		107%	102%	
Pedestrian areas North		118%				107%		
Pedestrian areas South 1		118%				107%		
Pedestrian areas South 2		118%				107%		
Boardwalk and lookout		118%			104%	107%	102%	
Signs and linemarking							102%	
Landscaping		118%	111%			107%	102%	
Access track and surrounds		118%	111%	107%		107%	102%	
Viaduct								
Piling		118%	111%	107%	104%		102%	
Abutments		116%	109%	104%	102%	104%		
Pile caps		116%	109%	104%	102%	104%	100%	
Piers		116%	109%	104%	102%	104%	100%	
Parapets and crash rails		116%		104%	102%	104%		
Expansion joint		116%	109%			104%		
Connecting slab		116%	109%	104%	102%	104%		
Area 2		118%	111%				102%	
Bridge								
Piling		118%	111%	107%	104%		102%	
Substructure		116%	109%	104%	102%	104%		
Pier columns		116%	109%	104%	102%	104%	100%	
Concrete		116%	109%	104%	102%	104%	100%	108%
Barriers		119%				107%		
Pedestrian barriers		119%				107%		
Expansion joint		116%				104%		
Connecting slab		116%	109%	104%	102%	104%		
Abutments		116%	109%	104%	102%	102%		
Superstructure								
Girders		116%	109%			104%		
Bearings		116%				104%		
Deck fit out		116%		104%	102%	104%		
Traffic barriers		119%				107%		
Pedestrian barriers		119%				107%		
Expansion joint		116%				104%		
Connecting slab 1		116%	109%	104%	102%	104%		
Connecting slab 2		116%	109%	104%	102%	104%		

## Step 5 – Finalise model

The model can now be finalised with a table that simply multiplies the estimate summary in **Figure 19** by the risk factor allocation in **Figure 20**. As the simulation runs, the risk factor distributions will be sampled, the results will be calculated in the table of combined risk factors and the effect of these on the cost will be calculated in the table where they are multiplied by the base estimate. The total and subtotal costs will be calculated there and recorded by the simulation tool.

**Figure 19: Final model – risk factor model 2.xlsx**

	Staff	Labour	Plant	Bulk materials			Subcon	Other	TOTAL
Sunk costs	-	-	-	-	-	-	-	-	-
Provisions	-	167,540	159,706	1,968,868	-	-	-	6,374,775	6,374,775
Establishment and preliminaries	-	-	-	-	-	-	-	893,734	2,929,452
Preliminaries time independent	9,210	87,392	72,471	462	-	143,627	609,685	-	922,847
Preliminaries time related	6,790,044	61,916	750,791	-	-	355,383	94,194	1,203,519	9,255,847
<b>Direct costs</b>									
<b>Area 1</b>									
Design	2,243,249	-	-	-	-	-	-	-	2,243,249
Special plant	-	2,872,195	2,031,390	-	-	566,661	-	-	5,470,246
Site equipment	-	1,094,816	558,816	-	-	187,771	11,440	548,932	2,401,774
Environmental management	247,121	35,869	-	-	-	66,654	-	-	349,645
Stormwater drainage	-	2,863	1,487	13,063	-	943	110,942	-	129,299
Earthworks	-	226,151	159,486	-	-	-	816,002	-	1,201,639
Shotcreting	-	-	-	-	-	-	938,073	-	938,073
Retaining walls	-	154,980	1,222	56,662	-	301,753	280,582	-	795,199
Pavement	-	19,905	15,204	-	-	39,734	333,702	-	408,544
Fencing	-	-	-	-	-	-	86,051	-	86,051
Barriers	-	5,286	2,750	-	-	-	85,837	-	93,873
<b>Paths</b>									
Shared path North	-	11,510	572	789	-	24,146	6,387	-	43,403
Shared path South	-	3,522	341	249	-	7,021	1,980	-	13,114
Pedestrian areas North	-	5,985	-	-	-	40,792	-	-	46,777
Pedestrian areas South 1	-	10,240	-	-	-	93,726	-	-	103,967
Pedestrian areas South 2	-	2,305	-	-	-	16,327	-	-	18,632
Boardwalk and lookout	-	49,161	-	-	70,175	109,312	74,461	-	303,108
Signs and linemarking	-	-	-	-	-	-	38,608	-	38,608
Landscaping	-	7,758	3,173	-	-	25,604	25,895	-	62,430
Access track and surrounds	-	1,878,386	1,195,394	6,736	-	1,207,111	48,735	-	4,336,363
<b>Viaduct</b>									
Piling	-	5,675	2,112	275,374	227,404	-	635,302	-	1,145,867
Abutments	-	126,522	11,447	48,273	34,325	16,943	-	-	237,510
Pile caps	-	747,668	148,205	290,669	883,333	75,449	21,075	-	2,166,400
Piers	-	3,227,717	186,114	1,333,255	2,177,636	1,601,148	1,837,861	-	10,363,732
Parapets and crash rails	-	208,993	-	87,535	101,813	1,055,777	-	-	1,454,119
Expansion joint	-	35,840	381	-	-	149,041	-	-	185,263
Connecting slab	-	20,244	274	11,227	12,579	2,463	-	-	46,786
<b>Area 2</b>									
Bridge	-	112,668	83,029	-	-	-	7,060	-	202,757
Piling	-	5,076	2,074	88,698	67,394	-	180,038	-	343,280
Substructure	-	118,605	2,084	49,423	35,713	15,883	-	-	221,709
Pier columns	-	391,525	99,743	337,046	405,531	58,610	22,064	-	1,314,518
Concrete	-	827,496	178,286	529,868	940,233	424,549	644,916	1,606	3,546,953
Barriers	-	20,026	-	-	-	155,854	-	-	175,880
Pedestrian barriers	-	9,980	-	-	-	77,081	-	-	87,062
Expansion joint	-	20,658	-	-	-	92,266	-	-	112,924
Connecting slab	-	22,209	284	9,731	18,864	2,421	-	-	53,509
Abutments	-	116,472	2,352	41,772	32,609	14,489	-	-	207,693
<b>Superstructure</b>									
Girders	-	2,017	742	-	-	137,243	-	-	140,002
Bearings	-	1,716	-	-	-	32,746	-	-	34,462
Deck fit out	-	50,201	-	9,513	24,722	19,502	-	-	103,938
Traffic barriers	-	2,797	-	-	-	25,188	-	-	27,984
Pedestrian barriers	-	1,379	-	-	-	10,553	-	-	11,932
Expansion joint	-	2,341	-	-	-	9,229	-	-	11,569
Connecting slab 1	-	32,603	626	22,763	34,273	2,951	-	-	93,216
Connecting slab 2	-	37,939	4,020	25,085	21,698	12,941	-	-	101,683
Noise walls required at southern end									-
Total	9,289,625	12,846,146	5,674,576	5,207,061	5,088,301	7,178,892	6,910,890	9,022,566	60,957,661

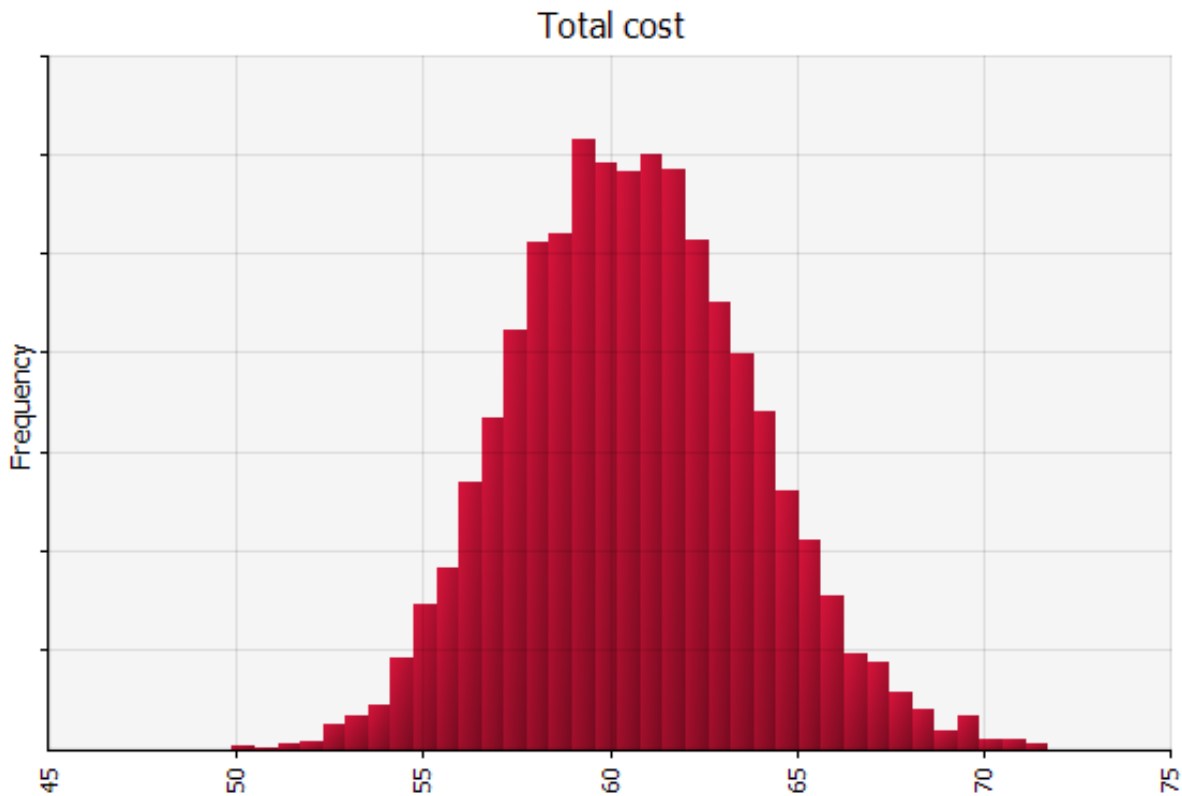
## 7. Analysis results

The results of a model can be presented either as graphs or numbers. It is common to make as much use of graphs as possible as they provide a quick and intuitive way of understanding the information. Because one purpose of risk modelling is generally to provide a basis for establishing contingency, results should be presented in such a way that helps decision-makers to set appropriate and realistic project budgets. Complex graphs and reams of statistical data is likely to leave the reader confused or bored (or both).

### Histogram plots

The histogram, or relative frequency, plot is the most commonly used in risk analysis as an initial presentation. A histogram is a bar chart with vertical bars representing the frequency with which a simulation's outcomes fell within the bar's span.

**Figure 20: Example histogram**



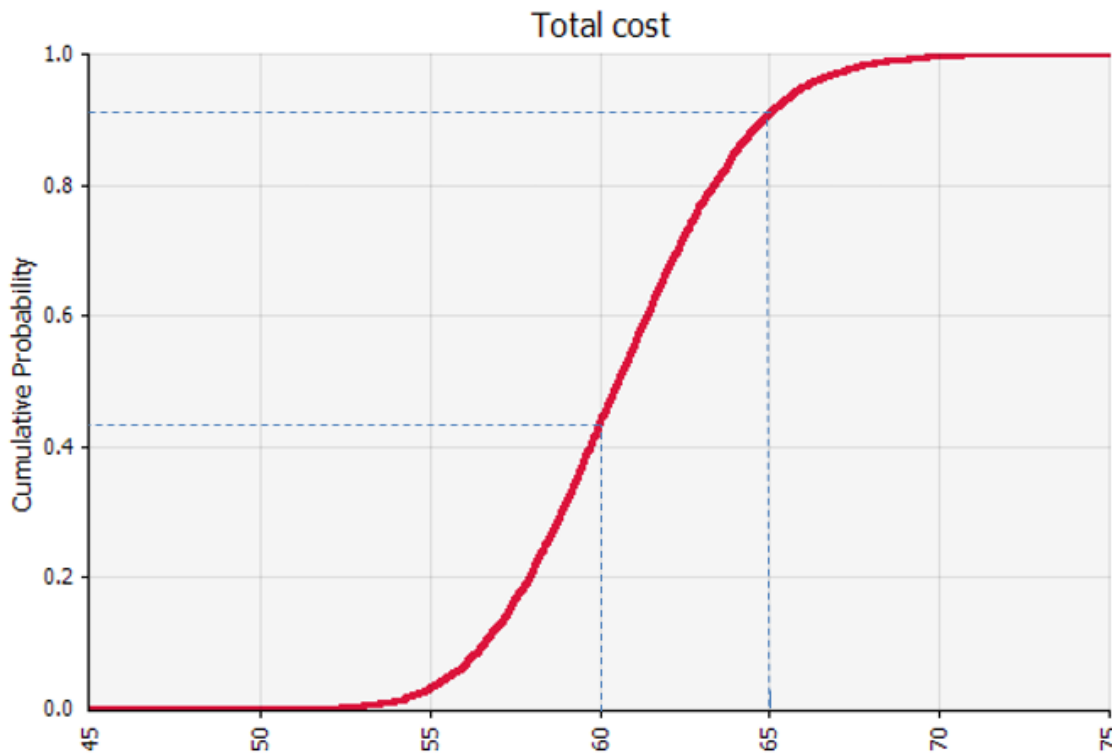
A histogram is useful in telling the risk “story.” **Figure 22** shows the story in terms of the mode (most likely outcome), the range (the difference between the best case and the worst case), and other features, if they exist, such as the skewness of the distribution, whether there are long tails, and/or whether there is any bimodality.

### Cumulative frequency plot or “S” curve

A cumulative probability diagram is based on the same information as a histogram. It has two forms, ascending and descending. The ascending plot is the most commonly used and shows the cumulative probability of being less than or equal to any x-axis value.



**Figure 21: Using the cumulative frequency plot to determine the probability of being between two values**



The “S” curve is useful for reading off quantitative information such as the probability of exceeding a particular value. It can also be used to find the probability of any outcome lying between two x-axis values, which is simply the difference between their respective cumulative probabilities. For example, from **Figure 23** it can be inferred that the probability of the project cost being between \$60m and \$65m is  $91\% - 44\% = 47\%$ .

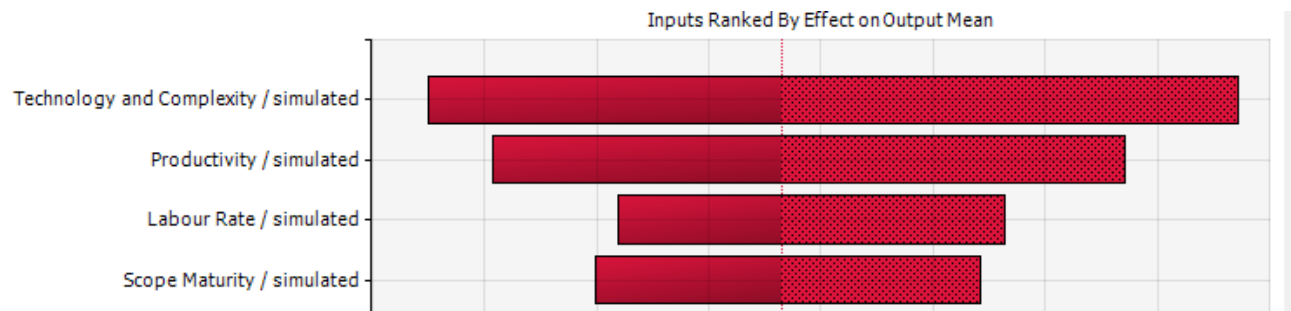
The limitation of “S” curves is that they do not display long tails and bimodality as clearly as histograms. However, they can be very useful to compare the risk profiles of multiple scenarios by overlaying them on the same chart.

## Tornado charts

Tornado charts are used to highlight the influence an input distribution has on the change in value of the output. In other words, they highlight the sensitivity of the outcome to the risks. Each input distribution is represented by a bar, and the horizontal range that the bars cover give some measure of the input distribution’s influence on the selected model output.

The main use of a tornado diagram is to identify the most influential model parameters. For example, **Figure 24** is intended to communicate to a manager that the contingency on this particular hypothetical project is very sensitive to Technology and Complexity, less so to risk factors such as Labour Rate and Scope Maturity.

**Figure 22: Example Tornado chart of selected project risk drivers**

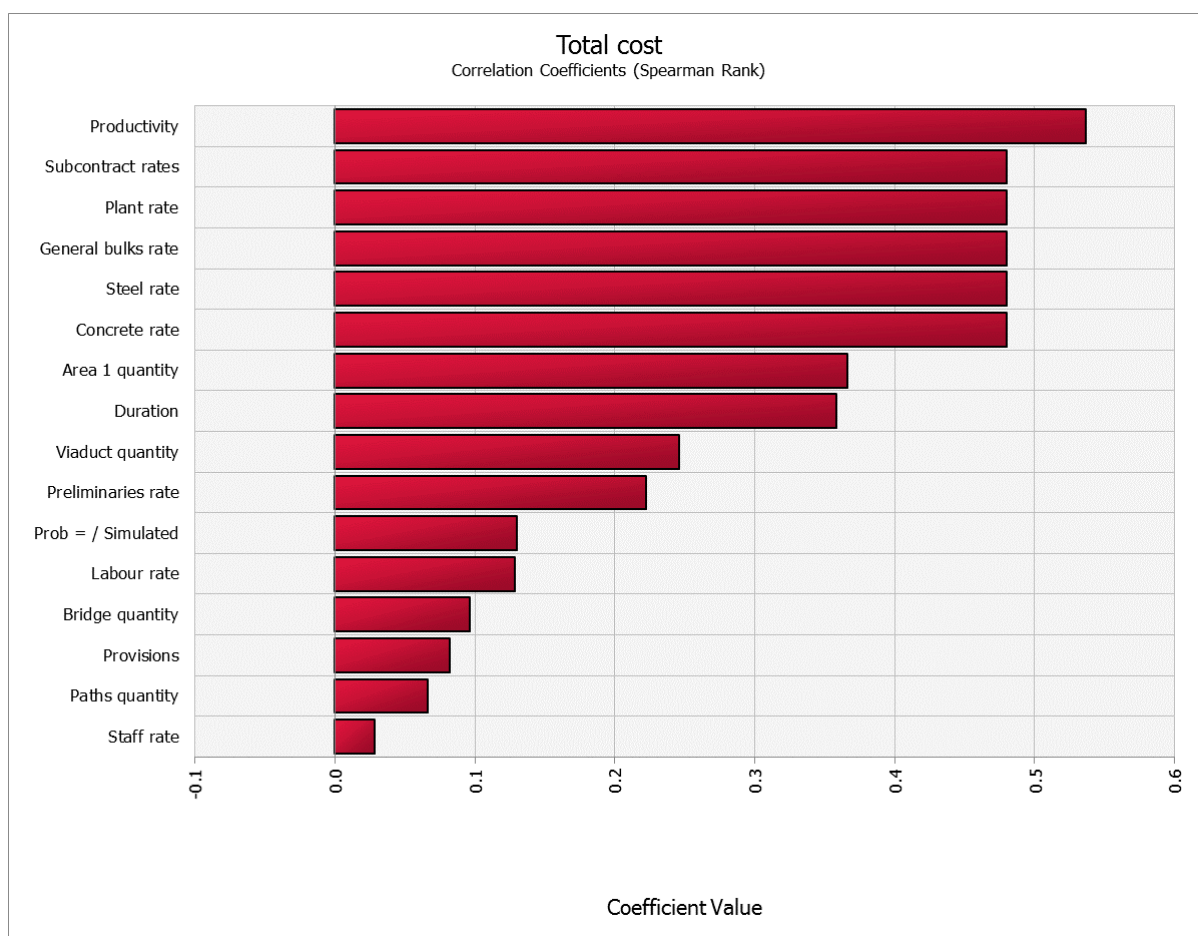


The tornado graph can be useful as a sanity check to verify that what the model is calculating as the most influential risks meets with reasonable expectations. If it indicates something is out of place, perhaps a factor being a lot more or a lot less important than expected, there may be a problem with the model. It is also possible that the expectation and associated modelled inputs were unrealistic and the model output is highlighting this.

**Figure 24** tends to be the most common form of tornado chart however most modelling tools include various sensitivity analysis features. For example, correlation sensitivity analysis calculates the correlation between the input values generated during a simulation and the output, the total cost. The higher this value, the larger the influence that source of uncertainty is having on the spread in the total cost.

A correlation sensitivity chart from <Risk factor model 1> is shown in **Figure 25**.

**Figure 23: Correlation sensitivity**



This shows that uncertainty about productivity is the largest cause of uncertainty in the total cost. Next is uncertainty in the group of rates that are expected to be correlated with one another by their common

dependence on market conditions. They have the same sensitivity rating because they are correlated with one another and move in unison during the simulation. Uncertainty about the bulk quantities in Area 1 is the next followed by uncertainty about the duration of the work. Compared to these factors, the remainder are relatively insignificant.

The length of the bars in **Figure 25** represent the correlation between variation in the factors and variation in the total cost through the many iterations of the Monte Carlo simulation. These values are very informative but cannot be used for any meaningful calculations in this context. The fact that one factor has a correlation half the value of another cannot be interpreted as meaning that it is half as important. The bars simply indicate the ranking of factors from most to least important.

Once a model is validated, the uncertainty sensitivity information shows where additional effort might allow the uncertainty in the total cost to be reduced and make the outcome more predictable. This might not result in a lower cost overall but could make it clearer what the cost will be if the work proceeds.

## 8. Additional models

“I can see that it works in practice, but does it work in theory?” – Garrett Fitzgerald, Prime Minister of Ireland 1981-1987

This section provides details of three additional modelling techniques that, depending upon project phase, the Department considers appropriate to use when developing estimates to accompany funding submissions. The three techniques are:

- Hybrid parametric/expected value method
- Risk Driver approach
- First Principles Risk Analysis (FPRA)

### 8.1. Hybrid parametric/expected value method

As the name implies, this approach combines two separate approaches: a parametric method to quantify systemic risks, and expected value to quantify project-specific risks. These are combined into one model to produce a probabilistic output for decision-making.

As discussed at [Section 2.2](#), it has been claimed<sup>31</sup> that systemic risks are best quantified using empirically validated parametric modelling. The disadvantage is that organisations may not initially have suitable historical data with which to develop parametric models. However, there are publicly available industry sources, including AACE International, that provide the basic relationships and functioning models as a starting point which can be built upon, calibrated to a particular organisation and improved over time. The example hybrid model that will be introduced shortly uses Hollmann’s spreadsheet (with permission) to quantify systemic risks. It is stressed that the example percentages in this model need to be further tested and calibrated by agencies by applying their own knowledge and reviewing the historical performance of their projects.

Note that even when data is available, the input table can only be completed after interviews and/or workshops with relevant subject matter experts, aided by prior project records where available.

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<sup>31</sup> Hollmann, J (2016) *Project Risk Quantification: A Practitioner’s Guide to Realistic Cost and Schedule Risk Management*, Probabilistic Publishing, Gainesville, FL

The parametric tool will provide a distribution (optimistic, most likely, pessimistic) for systemic risk contingency which is applied to the base estimate as one of the model inputs which is then assessed using Monte Carlo sampling.

## Using expected value for project-specific risks

Expected value is a technique that has been used for decision and risk management for decades. This guidance note will not describe the technique in detail other than to note the importance of screening for, and only including significant risks in the contingency calculation. Cluttering the analysis with minor risks presents a danger of double counting as the parametric model effectively takes account of run of the mill disturbances to project implementation.

## Screening

The expected value method builds on information that is already likely to exist in a risk register where the project team may have qualitatively ranked the risks using a traditional risk matrix. Screening involves making a quick estimate of the impact of each residual (treated) risk. If the calculation results in a value that is not significant in relation to the base estimate, then it is dropped from consideration (but kept in the register) and not used in the contingency calculation. Significance is judged using the criticality criteria as per AACE RP 41R-08 as follows:

Bottom Line Critical Variances		
Bottom line cost	Conceptual estimates (AACE Classes 3, 4, 5)	Detailed Estimates (AACE Classes 1, 2)
Cost Δ	± 0.5%	± 0.2%

Once the critical risks have been identified after screening, their values (probability of occurrence and impact if it occurs) are refined. It has been suggested that if more than about 15 residual risk are critical the screening has not been disciplined enough, and/or risk treatment has been ineffective<sup>32</sup>. This guidance note suggests that 20 project-specific risks should be considered to be the maximum for inclusion in the contingency calculation. Correlation between any risks must also be identified and modelled.

The screening approach has two major benefits, it:

- avoids the need to quantify a large number of project-specific risks.
- helps limits the number of inputs into the simulation model.

## Example parametric/risk-event model

The cost risk model inputs include:

- the base estimate.
- the parametric model outcome distribution (i.e. the systemic risk impact which is included as the first risk in the expected value tool).
- the products of the distribution of probability times the distribution of the cost impact for each project-specific risk.

A simple example model created using a hypothetical project is outlined below.

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<sup>32</sup> AACE (2011) Recommended Practice RP65R-11 Integrated Cost and Schedule Risk Analysis and Contingency Determination Using Expected Value

## Quantification of systemic risks

Where reliable records of organisational performance relating to systemic risks are not available, the Hollmann model is current and practical alternative. On an actual project, the inputs into the parametric model must be made after interviews/workshops with appropriate project personnel.

The input values for this example model (the yellow cells) for a hypothetical road project are based on the following assumptions:

- There is no new technology.
- Process severity is not applicable;
- Complexity is low given there are few projects interfaces or traffic management requirements.
- The team development, project control and estimate bias are fair.
- As a road project, equipment is generally not applicable (exceptions would include items such tolling systems, electrical equipment on rail projects, etc).
- 40% of the project by value will be on a firm, fixed price basis.
- Bias of the estimate is typical (i.e. biased, but not excessively over or under-biased).

**Figure 26** below shows the inputs and outputs of the parametric growth model.

Figure 24: Example parametric growth model

COST GROWTH			
RISK DRIVER	ENTER PARAMETER (a)	COEFFICIENT (b)	a*b
CONSTANT			-30.5
SCOPE	3		
PLANNING	3		
ENGINEERING	3		
SCOPE DEFINITION	3.0	9.8	29.4
NEW TECHNOLOGY	0%	0.12	0.0
PROCESS SEVERITY	0	1	0.0
COMPLEXITY	2	1.2	2.4
SUBTOTAL BASE (prior to adjustments)			1.3
ADJUSTMENTS	Complex Exec Strategy? >	No	
Team Development	Fair		0
Project Control	Fair		0
Estimate Basis	Fair		0
Equipment	0%		4
Fixed Price	40%		-3
TOTAL BASE (prior to bias adjustment; rounded to whole number)			2
Bias	Typical		0
SYSTEMIC COST CONTINGENCY (at shown chance of under run)			
MEAN			2%
10%			-5%
50%			1%
90%			9%

The systemic cost contingency percentage values that relate to 10%, 50% and 90% circled in the figure above) become input values for a triangular distribution applied to the base estimate in the model.

### Project specific risks

Following screening, the probability and the range of cost impacts of each critical risk is refined. **Figure 27** shows an example of the information captured for a typical risk event.

Figure 25: Typical risk event data

Project-Specific Risk Event		Probability (%)	Cost Impact range (thousands \$)			Expected value (prob x impact)
No.	Description		Low	Most Likely	High	
7	extreme weather event (storm, flood)	25%	\$ 100	\$ 250	\$ 400	\$ 63
Assumed Risk Response		Either accept (eg demobilise and allow to dry in case of rain) or mitigate (mobilise pumping, etc)				

### Simulation model

The simulation model is now created and run combining the systemic and project-specific risks. Correlation (if any) between project-specific risks should be accounted for using a correlation matrix. The example below (**Figure 28**) contains eight inputs: the distribution representing the systemic risks which is applied to the base estimate [Cell H5 = RiskTriang(E5,F5,G5)\*C2], and seven project-specific risks.

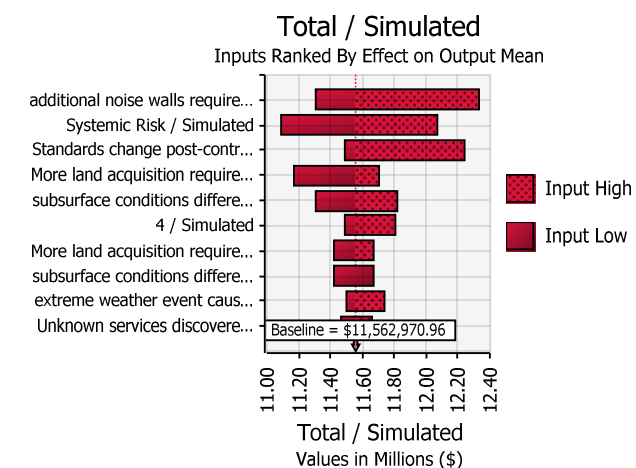
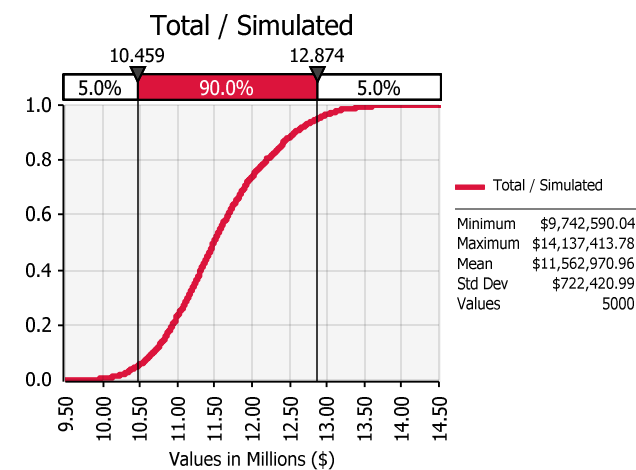
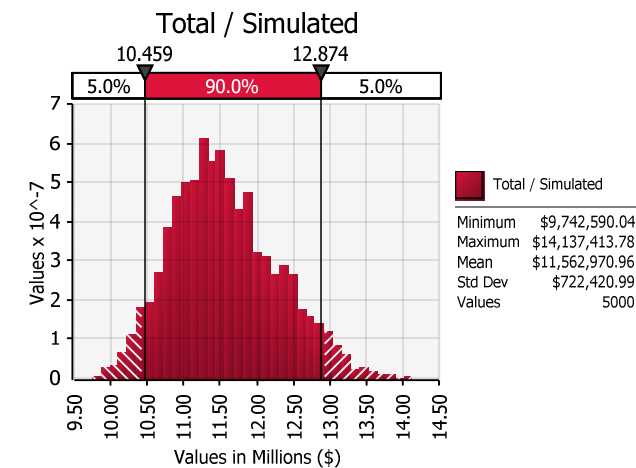
**Figure 26: Example simulation model**

	A	B	C	D	E	F	G	H
1								
2		<b>Hypothetical Road Project</b>						
3		<b>Base Estimate</b>	\$ 12,552,200					
4					<b>Cost Impact Range</b>			
5	<b>Risk</b>	<b>Description</b>		<b>Probability</b>	<b>Optimistic</b>	<b>Most Likely</b>	<b>Pessimistic</b>	<b>Simulated</b>
6	1	Systemic Risk		100%	-5%	1%	9%	\$ 209,203
7		<b>Project -Specific Risks</b>						
8	2	Unknown services discovered on site during construction		50%	\$ 100,000	\$ 150,000	\$ 250,000	\$ 83,333
9	3	Subsurface conditions different to assumed		50%	\$ 300,000	\$ 500,000	\$ 750,000	\$ 258,333
10	4	Existing material unsuitable for embankment requiring importation of fill		25%	\$ 150,000	\$ 300,000	\$ 500,000	\$ 79,167
11	5	Standards change post-contract award		10%	\$ 500,000	\$ 750,000	\$ 1,000,000	\$ 75,000
12	6	Additional noise walls required		25%	\$ 800,000	\$ 1,000,000	\$ 1,300,000	\$ 258,333
13	7	More land acquisition required than assumed		75%	\$ 400,000	\$ 500,000	\$ 800,000	\$ 425,000
14	8	Extreme weather event causes delays		25%	\$ 100,000	\$ 250,000	\$ 400,000	\$ 62,500
15								
16		<b>Total</b>						\$ 14,003,070

**Figure 29** below shows the output after the hybrid parametric/expected value simulation model has been un.



**Figure 27: Risk model outputs**



Simulation Summary Information	
Workbook Name	Hybrid parametric model.xlsx
Number of Simulations	1
Number of Iterations	5000
Number of Inputs	15
Number of Outputs	1
Sampling Type	Latin Hypercube
Simulation Start Time	
Simulation Duration	00:00:02
Random # Generator	Mersenne Twister
Random Seed	1517812708

Summary Statistics for Total / Simulated			
Statistics		Percentile	
Minimum	\$ 9,742,590	5%	\$ 10,459,481
Maximum	\$ 14,137,414	10%	\$ 10,688,387
Mean	\$ 11,562,971	15%	\$ 10,829,099
Std Dev	\$ 722,421	20%	\$ 10,940,618
Variance	5.21892E+11	25%	\$ 11,039,534
Skewness	0.379253478	30%	\$ 11,140,671
Kurtosis	2.811068529	35%	\$ 11,235,316
Median	\$ 11,489,323	40%	\$ 11,313,750
Mode	\$ 11,308,271	45%	\$ 11,406,919
Left X	\$ 10,459,481	50%	\$ 11,489,323
Left P	5%	55%	\$ 11,575,820
Right X	\$ 12,873,933	60%	\$ 11,674,217
Right P	95%	65%	\$ 11,784,563
Diff X	\$ 2,414,451	70%	\$ 11,897,488
Diff P	90%	75%	\$ 12,025,106
#Errors	0	80%	\$ 12,187,256
Filter Min	Off	85%	\$ 12,378,778
Filter Max	Off	90%	\$ 12,563,241
#Filtered	0	95%	\$ 12,873,933

Change in Output Statistic for Total / Simulated			
Rank	Name	Lower	Upper
1	additional noise	\$ 11,303,254	\$ 12,334,694
2	Systemic Risk / Si	\$ 11,091,554	\$ 12,069,484
3	Standards chang	\$ 11,487,263	\$ 12,244,347
4	More land acquis	\$ 11,168,684	\$ 11,702,112
5	subsurface condi	\$ 11,306,709	\$ 11,819,233
6	4 / Simulated	\$ 11,484,266	\$ 11,810,715
7	More land acquis	\$ 11,415,927	\$ 11,675,636
8	subsurface condi	\$ 11,418,728	\$ 11,669,608
9	extreme weathe	\$ 11,502,019	\$ 11,743,285
10	Unknown service	\$ 11,467,676	\$ 11,658,266

## 8.2. Risk driver method

A risk driver can be defined as: any root cause that may force a project to have outcomes different than the plan<sup>33</sup>.

If something is already causing a deviation from the plan, it is not a risk driver. It is a problem that must be dealt with. Conversely, if there is no chance of a certain situation arising on a particular project, it cannot be a risk driver (on that project).

David Hulett acknowledges that there are some claims that Monte Carlo simulation is unable to quantify the impact of systemic risks to project cost (and schedule). Hulett argues that Monte Carlo simulation methods (specifically the Risk Driver Method) can incorporate systemic risks as well as uncertainty and project specific risks in the one inclusive approach. The caveat is that while Hulett asserts that systemic risks can be incorporated without parametric methods two things are required:

- A rich database of project results on comparable projects.
- Assessment of the likelihood that the systemic risks will apply to the project being analysed<sup>34</sup>.

The risk driver approach starts with the risks that are typically prioritised in the risk register that is usually available. Similar to the risk factor approach, the risk driver approach assigns risks directly to the project element costs that they will affect if they occur. As with the risk factor approach, there are often risk drivers that are assigned to more than one cost element, and some cost elements will have more than one risk driver assigned. Hence correlation between risks is again implicitly accounted for.

Risks and uncertainties are typically worded in keeping with the definition of a risk driver. For example the systemic risk “*Level of Technology*” would be worded as “The project may use significant new technology”. It is thus stated in such a way as to be more amenable to assigning a probability of occurrence. It is suggested that even on very large and complex projects, there should be no more than 20-40 risk drivers<sup>35</sup>.

Once the risks have been identified a risk workshop/interview process is undertaken to determine:

- The probability that the risk may occur.
- The impact range if it were to occur.
- The project elements that it affects if it does occur.

The information collected regarding probability and impact for the risks will typically be similar to the abbreviated **Table 13** below<sup>36</sup>.

**Table 13: Risk driver probability table**

Risk Driver	Probability	Risk Factor		
		Optimistic	Most likely	Pessimistic
Driver A	30%	1.00	1.05	1.10
Driver B	60%	1.00	1.10	1.30
Driver C	100%	0.95	1.00	1.10
...etc	...	...	...	...

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<sup>33</sup> Stump, E., (2000), *The Risk Driver Impact Approach to Estimation of Cost Risks: Clear Thinking About Project Cost Risk Analysis* (downloadable at [http://galorath.com/wp-content/uploads/2014/08/stump\\_risk\\_driver\\_approach.pdf](http://galorath.com/wp-content/uploads/2014/08/stump_risk_driver_approach.pdf))

<sup>34</sup> Hulett D, Whitehead W, (Risk 2142) The Monte Carlo Method for Modelling and Mitigating Systemic Risk

<sup>35</sup> Hulett D, & Norbisch M, (2012) Integrated Cost-Schedule Risk Analysis

<sup>36</sup> Hulett D, (2011) Integrated Cost-Schedule Risk Analysis, Gower Publishing Limited, Farnham England

The risk drivers are then assigned to either cost elements, or to project activities (typically when utilising the approach for integrated cost-schedule risk analysis) at a summary level. **Table 14** shows risk drivers assigned to costs at an aggregated level.

**Table 14: Cost elements affected by risk drivers**

Cost Element	Risks			
	Driver A	Driver B	Driver C	...etc
Preliminaries	X		X	
Earthworks		X		
...		X		X
...	X		X	

### Iteration

The mechanics of setting up and running the simulation are essentially the same as for the risk factor method.

### Applications

The risk driver approach has found application on projects, particularly in the oil and gas and process-plant industries for which a significant proportion of the cost is related to equipment. Because it is somewhat premised on the assumption that the risk in the project can be described by several key strategic risks which have a certain probability of occurrence, it is well-suited to projects for which key risks may be those such as:

- Fabrication and installation productivity rates.
- Cost variability and/or availability of long-lead equipment.
- Installation delays or issues due to coordination and/or commissioning problems.

However, the Department considers the approach is flexible enough to use on land transport infrastructure projects.

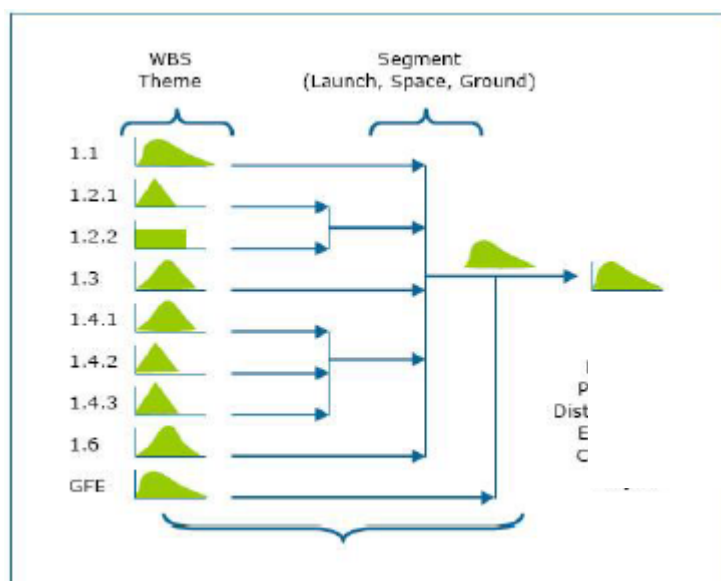
## 8.3. First Principles Risk Analysis

When a FPRA model is used to perform a cost risk analysis, an analyst needs to know the uncertainty of the individual cost estimates, their statistical dependencies, and how to calculate their sums. **Table 15** summarises the most common challenges associated with the commonly used 3-Point estimating (line item ranging) methods and how FPRA attempts to address them. Note that for the purposes of FPRA, risks are classified as being either inherent or contingent.

**Table 15: First Principles Risk Analysis**

Challenges associated with 3-point Estimate method	How FPRA is trying to address it	Important notes for appropriate application of FPRA
Selection of appropriate level of WBS for allocating 3-Point ranges (noting that traditionally, all cost items within the	By allocating the ranges at the lowest meaningful WBS level against appropriate first principle component's of cost item, i.e. Labour,	<ul style="list-style-type: none"> <li>- Not every single cost line item at the lowest level should be ranged.</li> <li>- Not every single cost line item will have all components of L/M/P/SC, for example installation of utilities might have only 'M' and 'SC'.</li> <li>- Not each element of cost, i.e. L/M/P/SC at the cost line item should be ranged. For example in the installation of utilities with</li> </ul>

Challenges associated with 3-point Estimate method	How FPRA is trying to address it	Important notes for appropriate application of FPRA
estimate are ranged)	<p>Material, Plant, and Sub-Contract.</p> <p>The results should be then rolled and aggregated up to higher reasonable WBS levels for simulation modelling purposes.</p>	<p>‘M’ and ‘SC’, while there is a range for number of utilities ‘M’, if we are subcontracting installation on a lump sum contract, there is no need for a range on ‘SC’. However, due to uncertainties associated with underground services, there is a possibility of additional claim or delay by this contractor which should be assessed and modelled as a ‘contingent risk’.</p> <ul style="list-style-type: none"> <li>- No pre-defined level of WBS to assess the ranges. Similar to cost allocation for Earned Value implementation and as a bottom-up approach, different levels of WBS may be selected.</li> <li>- Uncertainty ranges for cost items against labour, plant, material and subcontract are not simply added together to create an aggregate range for that cost item; the method is used to inform and improve the quality of ranges for subjective uncertainties before simulation at a higher level.</li> <li>- The illustration below<sup>37</sup> provides a visual representation of the process.</li> </ul>



Understanding and appropriately modelling correlations between inherent risks to address the relationships	<p>- By using the Functional Correlation (Implicit) for all key dependent cost variables. For example if design cost is a function (percentage) of</p>	<p>- In the absence of objective data, risk analysts and estimators should make a subjective assessment of correlation.</p> <p>- Apply functional correlations within the risk model whenever possible.</p> <p>- Pragmatically it is recommended that approximate correlation coefficients between appropriate WBS elements be applied. This can be as simple as determining whether two WBS elements are correlated by a small amount or by a large amount and whether</p>
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<sup>37</sup> NASA (2015) NASA Cost Estimating Handbook Version 4.0, Appendix G

Challenges associated with 3-point Estimate method	How FPRA is trying to address it	Important notes for appropriate application of FPRA
between the model inputs	<p>'Direct' cost, this should be functionally linked</p> <p>- By using Applied Correlation (Explicit)</p>	that correlation is positive or negative. These WBS elements should be aligned with selected WBS levels for simulation.
Understanding and appropriately modelling correlations between contingent risks	By using Applied Correlation (Explicit)	Correlations between contingent risks should be also assessed and modelled.
Assessment and visibility of key areas of uncertainty	By allocating the ranges against applicable components of L/M/P/SC at the lowest appropriate WBS level, then rolling them up to higher levels of WBS for simulation purposes.	<p>Further alignment between cost estimators, schedulers and risk analysts. The objective of method is not just to generate a contingency number but also to facilitate right conversations between the team to discuss, assess, and mitigate key areas of uncertainties and risks as much as possible.</p> <p>The residual risks/uncertainties (depending on delivery strategies) should be then modelled for assessing the optimum contingency.</p>
Cognitive Biases	As above.	As Above.

The steps below represent the general process for undertaking FPRA:

1. Develop a first principles cost estimate (base estimate), representing the most likely assumptions, structured against Labour, Plant, Material and Sub-contract as per **Figure 30**.

Project Cost Estimate (Base Estimate)						
WBS Code	Cost Item Description	Labour \$	Plant \$	Material \$	Subcontract \$	Base Estimate \$
1.1	CONTRACTORS COSTS					
1.1.1	CONTRACTORS DESIGN COSTS/FEES				\$24,560,000	\$ 24,560,000
1.1.2	CONTRACTORS PRELIMINARIES	\$11,914,462	\$3,458,550	\$5,041,101	\$6,016,086	\$ 26,430,199
1.2	RAIL EXTENSION					
1.2.1	EARTHWORKS DRAINAGE AND FENCING	\$10,494,514	\$19,727,632	\$3,546,677	\$3,299,692	\$ 37,068,515
1.2.2	RETAINING WALLS	\$3,235,847	\$3,249,640	\$3,498,750	\$10,517,470	\$ 20,501,707
1.2.3	PSP	\$2,227,128	\$6,340,202	\$5,288,492	\$8,921,296	\$ 22,777,118
1.3	NOISE AND VIBRATION					
1.3.1	NOISE WALLS				\$7,827,500	\$ 7,827,500
1.3.2	BALLAST MATTING	\$1,177,729	\$2,153,430	\$5,819,604		\$ 9,150,763
1.4	STRUCTURES					
1.4.1	P. BRIDGE	\$1,518,769	\$1,294,149	\$1,395,990	\$1,967,950	\$ 6,176,858
1.4.3	R. RD BRIDGE					
1.4.3.1	TEMPORARY DIVISION AND PERMANENT WORKS	\$1,138,410	\$1,414,813	\$2,112,851	\$179,218	\$ 4,845,292
1.4.3.2	EXCAVATION	\$2,137,180	\$1,352,904	\$1,295,895	\$2,282,921	\$ 7,068,900
1.4.3.3	CONCRETE PLACEMENT				\$7,622,426	\$ 7,622,426
1.4.7	B. RD					
1.4.7.3	CONCRETE PLACEMENT				\$4,574,787	\$ 4,574,787
1.4.7.4	BRIDGE RAILINGS	\$2,230,648	\$211,379	\$1,100,154	\$156,352	\$ 3,698,532
1.4.7.5	ROAD FITTINGS AND FIXTURES	\$1,130,648	\$111,379	\$1,100,154	\$256,352	\$ 2,598,532
1.5	TRACKWORK	\$3,151,722	\$4,343,733	\$5,569,023	\$5,278,728	\$ 18,343,206
1.6	OHLE	\$3,728,088	\$5,950,734	\$5,638,127	\$6,382,126	\$ 21,699,075
1.7	STATIONS					
1.7.1	A. BRIDGE	\$1,520,960	\$713,795	\$1,597,151	\$15,463,951	\$ 19,295,857
1.7.2	E. BRIDGE	\$1,676,776	\$1,340,230	\$1,246,976	\$12,450,004	\$ 16,713,986
1.8	TUNNELS	\$3,314,235	\$1,548,277	\$2,968,229	\$11,150,466	\$ 18,981,207
1.9	SIGNALLING AND COMMUNICATIONS				\$25,448,203	\$ 25,448,203
1.10	CONTRACTORS CONSTRUCTION COSTS EXCL. MARGIN					
1.10.1	CONTRACTORS PROFIT MARGIN	\$6,213,919	\$4,692,319	\$6,193,486	\$15,875,863	\$ 32,975,587
1.11	CONTRACTORS CONSTRUCTION COSTS INCL. MARGIN					
1.12	OWNERS/CLIENT COSTS					
1.12.1	OWNERS PROFESSIONAL FEES AND COSTS			\$14,353,618		\$ 14,353,618
1.12.2	ADVANCE WORKS				\$1,450,292	\$ 1,450,292
1.12.3	LAND COSTS			\$12,121,979		\$ 12,121,979
1.12.4	Traction Power				\$8,362,978	\$ 8,362,978
	Total Cost Excl. Contingency and Escalation	\$ 56,811,034	\$ 57,903,165	\$ 79,888,257	\$ 180,044,661	\$ 374,647,117
	Cost Estimate Split % (L, P, M, SC)	15.16%	15.46%	21.32%	48.06%	100.00%

**Figure 28: Aggregated first principles estimate**

- Identify and assess main areas of uncertainties, i.e. ‘inherent risks’ within the cost estimate. These main uncertainties should be identified at applicable cost items against ‘labour’ and/or ‘plant’ and/or ‘material’ and/or ‘sub-contract’ at the lowest level/s which they can be reasonably and accurately be assessed.
- Assess and quantify the main uncertainty ranges at the appropriate level. **Figure 31** presents these ranges for labour at aggregated level and includes a short commentary explaining the basis of range for each item.

**Figure 31: Determining labour rate ranges against aggregated cost elements**

WBS Code	Cost Item Description	Labour \$	Notes/Basis of Valuation	Best Case %	Most Likely %	Worst Case %
1.1	<b>CONTRACTORS COSTS</b>					
1.1.1	CONTRACTORS DESIGN COSTS/FEES					
1.1.2	CONTRACTORS PRELIMINARIES	\$11,914,462	Range of Preliminaries is dependent on complexity, duration of the project and physical works. Market range is 20% to 25% of Direct Cost. The cost allowance is at the low end of the range. (Optimistic)	-3%	0%	12%
1.2	<b>RAIL EXTENSION</b>					
1.2.1	EARTHWORKS DRAINAGE AND FENCING	\$10,494,514	Method of pricing: First Principle, Conservative estimate	-15%	-8%	3%
1.2.2	RETAINING WALLS	\$3,235,847	Method of pricing: Rates based estimate, Conservative estimate	-20%	-10%	3%
1.2.3	PSP	\$2,227,128	Method of pricing: Rates based estimate, Reasonable Estimate	-7%	0%	13%
1.3	<b>NOISE AND VIBRATION</b>					
1.3.1	NOISE WALLS					
1.3.2	BALLAST MATTING	\$1,177,729	Method of pricing: First Principle, Conservative estimate. Low productivity rate allows 43 extra days than the standard productivity rate of 25 sqm/hour.	-7%	0%	10%
1.4	<b>STRUCTURES</b>					
1.4.1	P. BRIDGE	\$1,518,769	High degree of confidence in Qtys and Rates	-5%	0%	10%
1.4.3	R. RD BRIDGE					
1.4.3.1	TEMPORARY DIVISION AND PERMANENT WORKS	\$1,138,410	Most Likely estimate	-10%	0%	20%
1.4.3.2	EXCAVATION	\$2,137,180	Geotechnical Investigation not yet complete.	-10%	0%	20%
1.4.3.3	CONCRETE PLACEMENT		Contract type			
1.4.7	B. RD					
1.4.7.3	CONCRETE PLACEMENT		Contract type			
1.4.7.4	BRIDGE RAILINGS	\$2,230,648	Design change	-5%	0%	15%
1.4.7.5	ROAD FITTINGS AND FIXTURES	\$1,130,648	Design change	-5%	0%	15%
1.5	TRACKWORK	\$3,151,722	Productivity for trackwork could be 450m to 600m per shift. Assumed productivity is low. Conservative estimate	-30%	-20%	10%
1.6	OHLE	\$3,728,088	The overall productivity 202 m/day is low and the estimate is conservative.	-25%	-15%	10%
1.7	<b>STATIONS</b>					
1.7.1	A. BRIDGE	\$1,520,960	Design Progress	-10%	-5%	15%
1.7.2	E. BRIDGE	\$1,676,776	Method of pricing: First Principle, Most Likely estimate	-10%	-5%	15%
1.8	<b>TUNNELS</b>	\$3,314,235	Design Progress	-10%	0%	15%
1.9	<b>SIGNALLING AND COMMUNICATIONS</b>					
1.10	<b>CONTRACTORS CONSTRUCTION COSTS EXCL. MARGIN</b>					
1.10.1	CONTRACTORS PROFIT MARGIN	\$6,213,919	10% (current estimate), 12.5% (ML), 14% (WC)			
1.11	<b>CONTRACTORS CONSTRUCTION COSTS INCL. MARGIN</b>					
1.12	<b>OWNERS/CLIENT COSTS</b>					
1.12.1	OWNERS PROFESSIONAL FEES AND COSTS					
1.12.2	ADVANCE WORKS					
1.12.3	LAND COSTS					
1.12.4	Traction Power					
	<b>Total Cost Excl. Contingency and Escalation</b>	<b>\$ 56,811,034</b>				
	<b>Cost Estimate Split % (L, P, M, SC)</b>	<b>15.16%</b>				

- Account for dependencies between dependent cost estimates, e.g. if cost of design has been estimated by using 8% of direct cost, this dependency should be established within the FPRA risk model using functional or structural links.
- Aggregate these ranges to the reasonable higher levels of CBS/WBS, e.g. Level 4 or 5 for simulation modelling. Aggregating ranges for labour, plant, material and subcontract for each cost line item should not be achieved by simply adding them together, which is theoretically and mathematically incorrect, or rolling up the ranges to a higher level. However, the expectation is that because the ranges have been allocated at the lowest reasonable level of WBS against Labour, Plant, Material and Subcontract, estimators will be able to form a realistic judgement of the possible overall range for that line item (**Figure 32**).



**Figure 29: Overall ranges for aggregated cost elements**

Project Cost Estimate (Base Estimate)		Inherent Ranges			OUTPUT 1 (Inherent)  Simulation @ Total Range level
WBS Code	Cost Item Description	Best Case \$	Most Likely \$	Worst Case \$	
1.1	CONTRACTORS COSTS				
1.1.1	CONTRACTORS DESIGN COSTS/FEES	\$ 24,433,639	\$ 29,320,367	\$ 34,207,095.04	\$ 29,320,367
1.1.2	CONTRACTORS PRELIMINARIES	\$ 25,637,293	\$ 26,430,199	\$ 29,601,823	\$ 26,430,199
1.2	RAIL EXTENSION				
1.2.1	EARTHWORKS DRAINAGE AND FENCING	\$ 32,482,270	\$ 34,722,920	\$ 38,712,050	\$ 34,722,920
1.2.2	RETAINING WALLS	\$ 16,927,239	\$ 18,977,410	\$ 21,462,075	\$ 18,977,410
1.2.3	PSP	\$ 21,593,720	\$ 22,366,118	\$ 25,024,440	\$ 22,366,118
1.3	NOISE AND VIBRATION	\$ 16,497,170	\$ 16,818,802	\$ 17,893,339	\$ 16,818,802
1.3.1	NOISE WALLS	\$ 7,827,500	\$ 7,827,500	\$ 7,827,500	
1.3.2	BALLAST MATTING	\$ 8,669,670	\$ 8,991,302	\$ 10,065,839	
1.4	STRUCTURES	\$ 34,847,818	\$ 36,097,820	\$ 39,444,328	\$ 36,097,820
1.4.1	P. BRIDGE	\$ 5,868,015	\$ 6,083,696	\$ 6,654,801	
1.4.3	R. RD BRIDGE				
1.4.3.1	TEMPORARY DIVISION AND PERMANENT WORKS	\$ 4,440,464	\$ 4,707,769	\$ 5,395,841	
1.4.3.2	EXCAVATION	\$ 6,741,036	\$ 7,023,241	\$ 7,656,140	
1.4.3.3	CONCRETE PLACEMENT	\$ 7,241,305	\$ 7,469,978	\$ 8,155,996	
1.4.7	B. RD				
1.4.7.3	CONCRETE PLACEMENT	\$ 4,574,787	\$ 4,574,787	\$ 4,574,787	
1.4.7.4	BRIDGE RAILINGS	\$ 3,513,605	\$ 3,669,174	\$ 4,135,881	
1.4.7.5	ROAD FITTINGS AND FIXTURES	\$ 2,468,605	\$ 2,569,174	\$ 2,870,881	
1.5	TRACKWORK	\$ 14,359,393	\$ 16,953,287	\$ 19,721,782	\$ 16,953,287
1.6	OHLE	\$ 17,789,499	\$ 19,959,406	\$ 23,329,853	\$ 19,959,406
1.7	STATIONS	\$ 33,844,909	\$ 35,070,981	\$ 39,770,714	\$ 35,070,981
1.7.1	A. BRIDGE	\$ 18,175,159	\$ 18,816,397	\$ 21,301,491	
1.7.2	E. BRIDGE	\$ 15,669,750	\$ 16,254,584	\$ 18,469,223	
1.8	TUNNELS	\$ 17,421,201	\$ 18,430,409	\$ 21,045,039	\$ 18,430,409
1.9	SIGNALLING AND COMMUNICATIONS	\$ 24,175,793	\$ 24,939,239	\$ 28,501,987	\$ 24,939,239
1.10	CONTRACTORS CONSTRUCTION COSTS EXCL. MARGIN				
1.10.1	CONTRACTORS PROFIT MARGIN	\$ 30,008,695.93	\$ 37,510,869.91	\$ 42,012,174.30	\$ 37,510,870
1.11	CONTRACTORS CONSTRUCTION COSTS INCL. MARGIN				
1.12	OWNERS/CLIENT COSTS	\$ 34,546,938	\$ 36,288,867	\$ 40,190,873	\$ 36,288,867
1.12.1	OWNERS PROFESSIONAL FEES AND COSTS	\$ 13,635,937	\$ 14,353,618	\$ 15,788,980	
1.12.2	ADVANCE WORKS	\$ 1,450,292	\$ 1,450,292	\$ 1,450,292	
1.12.3	LAND COSTS	\$ 11,515,880	\$ 12,121,979	\$ 13,334,177	
1.12.4	Traction Power	\$ 7,944,829	\$ 8,362,978	\$ 9,617,425	
	Total Cost Excl. Contingency and Escalation	\$ 340,240,873	\$ 370,373,283	\$ 421,713,346	

6. Assess and apply correlations between various cost elements at the aggregated level. In the absence of objective data the process below is recommended when using FPRA.
  - a. Apply functional correlations within the risk model whenever possible.
  - b. Measure the correlation present in the model due to functional correlations and identify those elements with a low level of correlations, e.g. equal or less than 0.3.
    - i. Determine if specific elements that are currently uncorrelated should 'move together', that is, be correlated either negatively or positively.
  - c. Assign additional correlation using a correlation value between -1 and +1 at appropriate level of CBS/WBS, i.e. the aggregated level.
  - d. Measure and review the correlations again to ensure elements are properly correlated (**Figure 33**).

**Figure 30: Correlation matrix for inherent risk**

@RISK Correlations	CONTRACTORS DESIGN COSTS/FEES in \$AM\$8	CONTRACTORS PRELIMINARIES in \$AM\$9	EARTHWORKS DRAINAGE AND FENCING in \$AM\$11	RETAINING WALLS in \$AM\$12	PSP in \$AM\$13	NOISE AND VIBRATION in \$AM\$14	STRUCTURES in \$AM\$17	TRACKWORK in \$AM\$27	OHLE in \$AM\$28	STATIONS in \$AM\$29	TUNNELS in \$AM\$32	SIGNALLING AND COMMUNICATIONS in \$AM\$33	CONTRACTORS PROFIT MARGIN in \$AM\$35	OWNERS/CLIENT COSTS in \$AM\$37
CONTRACTORS DESIGN COSTS/FEES in \$AM\$8	1													
CONTRACTORS PRELIMINARIES in \$AM\$9	0	1												
EARTHWORKS DRAINAGE AND FENCING in \$AM\$11	0	0.7	1											
RETAINING WALLS in \$AM\$12	0	0.7	0.3	1										
PSP in \$AM\$13	0	0.7	0.3	0.5	1									
NOISE AND VIBRATION in \$AM\$14	0	0.7	0.5	0.3	0.3	1								
STRUCTURES in \$AM\$17	0	0.7	0.5	0.3	0.3	0.3	1							
TRACKWORK in \$AM\$27	0	0.7	0.5	0.3	0.3	0.3	0.5	1						
OHLE in \$AM\$28	0	0.7	0.5	0.3	0.3	0.3	0.3	0.3	1					
STATIONS in \$AM\$29	0	0.7	0.5	0.3	0.3	0.3	0.5	0.3	0.3	1				
TUNNELS in \$AM\$32	0	0.7	0.5	0.3	0.3	0.3	0.5	0.3	0.5	0.3	1			
SIGNALLING AND COMMUNICATIONS in \$AM\$33	0	0.7	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.3	0.5	1		
CONTRACTORS PROFIT MARGIN in \$AM\$35	0	0	0	0	0	0	0	0	0	0	0	0	1	
OWNERS/CLIENT COSTS in \$AM\$37	0	0.7	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1

7. Identify residual contingent risks (Figure 34).

**Figure 31: Contingent risks**

Contingent Risks									
Risk ID	Risk Description	Notes Basis of Valuation	Likelihood %	Simulated Probability	Residual Cost Consequence			Simulated Cost Consequence	OUTPUT 2 (Contingent)
					Best Case \$	Most Likely \$	Worst Case \$		
R1	Contractor may not be financially viable.	The costs associated with replacing the contractor. Project time delay impact included in R6	1%	0.01	\$ 30,000,000	\$ 50,000,000	\$ 100,000,000	\$ 66,130,663	\$ 661,307
R2	Shotcrete and rock bolts are required.	bolting as part of bulk earth works.	25%	0.25	\$ 1,000,000	\$ 3,000,000	\$ 10,000,000	\$ 5,500,045	\$ 1,375,011
R3	Substation is required to be built as part of the project.	Costs associated with additional materials.	1%	0.01	\$ 35,000,000	\$ 55,000,000	\$ 65,000,000	\$ 53,809,551	\$ 538,096
R4	CCTV may be required to be fixed to radio towers to view the rail corridor.	Additional cost of \$57m to bring in 132kV transmission, plus substation equipment.	50%	0.5	\$ 300,000	\$ 500,000	\$ 1,000,000	\$ 661,307	\$ 330,653
R5	New digital radio system is not available when required.	Additional design and construction costs.	5%	0.05	\$ 10,000,000	\$ 15,000,000	\$ 25,000,000	\$ 17,931,220	\$ 896,561
R6	Delay in Project - Schedule risk impact	Must install current analogue radio system and eventually digital system (when available) and perform cutover.							
		Base Schedule represents most likely conditions. The results of a separate schedule risk assessment have been costed for P10, P50 and P90 cases. Cost impact of the schedule risk is calculated based on the SRS results and the burn rate.							
		P10 = 26 days, P50 = 78 days & P90 = 155 days with the burn rate of \$63k per day.	100%	1	\$ 2,372,650	\$ 7,117,950	\$ 14,144,643	\$ 8,822,704	\$ 8,822,704
R7	Opportunity - to engage a top Tier 2 contractor	Current negotiations with XYZ Contractor	50%	0.5	-\$ 4,000,000	-\$ 2,000,000	-\$ 1,000,000	-\$ 2,119,045	-\$ 1,059,522
									\$ 11,564,809
									OUTPUT 2 (Contingent)

8. Assess and define correlation/s between the contingent risks (Figure 35).

**Figure 32: correlation matrix for contingent risks**

@RISK Correlations	Contractor may not be financially viable. in \$I\$6	Shot crete and rock bolts are required. in \$I\$7	Substation is required to be built as part of the project. in \$I\$8	CCTV may be required to be fixed to radio towers to view the rail corridor. in \$I\$9	New digital radio system is not available at Yanchep Rail Extension when required. in \$I\$10	Delay in Project - Schedule risk Impact in \$I\$11
Contractor may not be financially viable. in \$I\$6	1					
Shot crete and rock bolts are required. in \$I\$7	0	1				
Substation is required to be built as part of the project. in \$I\$8	0	0	1			
CCTV may be required to be fixed to radio towers to view the rail corridor. in \$I\$9	0	0	0	1		
New digital radio system is not available when required. in \$I\$10	0	0	0	0	1	
Delay in Project - Schedule risk Impact in \$I\$11	0.7	0.7	0.7	0.7	0.7	1

9. Run the Monte Carlo simulation.
10. Review, validate and finalise the results.

# Appendix A – Deriving risk factors

It is important to pay attention to the scale or materiality of any detail that is introduced into a model. There may be a difficult part of a project that is subject to unusual sources of risk that make it very interesting but, if its total potential impact on the project is a very small fraction of a percentage point, it probably does not warrant separate attention. The aim is to have as little detail as possible but as much as is necessary. Excessive detail simply results in parts of the model being correlated in ways that are very hard to analyse and represent realistically, as discussed in [Section 6.5](#).

A source of cost uncertainty should not usually be included as a separate item if:

- Its total impact is small.
- Its likelihood is very low (see [Section 4.4](#)).
- It is of the same general order of magnitude as a number of other sources that can be covered as a group and assessed in terms of their aggregate effect (see [Section 4.3](#)).

Reasons to include a source of uncertainty as a separate item include:

- When it is large, has a moderate to high likelihood of occurring and has to be incorporated into the general contingency, although see [Section 4.4](#) concerning risks that are too large to be encompassed in a general contingency.
- It represents a cost that is unrelated to any of the base estimate items, such as a fine or a requirement to provide community infrastructure off-site as part of an arrangement with the local community that is not expected to be incurred but might be forced on the project.

Leaving aside risk events, which are often not required to model project cost uncertainty, it is possible to describe three principles for decomposing the overall uncertainty in a part of the cost.

**Table A 1: Decomposing uncertainty into component parts**

Principle	Description
1	If there are two or more significant and distinct sources of uncertainty at work that affect a part of the cost, it will be easier to think clearly about them if they are addressed separately: e.g. uncertainty in quantities and uncertainty in unit rates or uncertainty in bulk material cost and uncertainty in the labour required to install or erect the materials.
2	If there are two or more examples of the same source of uncertainty and they have different characteristics they may need to be separated to allow realistic modelling: e.g. uncertainty in precast and in-situ concrete quantities or uncertainty in green field and brown field labour productivity.
3	If uncertainty in one or more components also affects other costs and will create a correlation between these costs if it is not broken out: e.g. uncertainty in labour rates that affects all direct labour.

If the participants in an analysis fall into a discussion about, say, a third of a cost's value having one set of characteristics and the remainder having different characteristics, it might be worth considering breaking the two apart in the cost structure and having a different factor for each one. At the same time, but separately, if a significant driver of one cost is also a significant driver of another cost then, unless it is the only driver of each of the costs, it probably need to be broken out as a separate factor in the model. This is illustrated in **Figure A1** and **Figure A2**, where the numbers next to factors refer to the three principles in **Table A1** and discussed in tables A2 and A3. The rationale for the breakdown is set out step by step in **Table A2**.

Figure A 1: Deriving risk factors for concrete structures

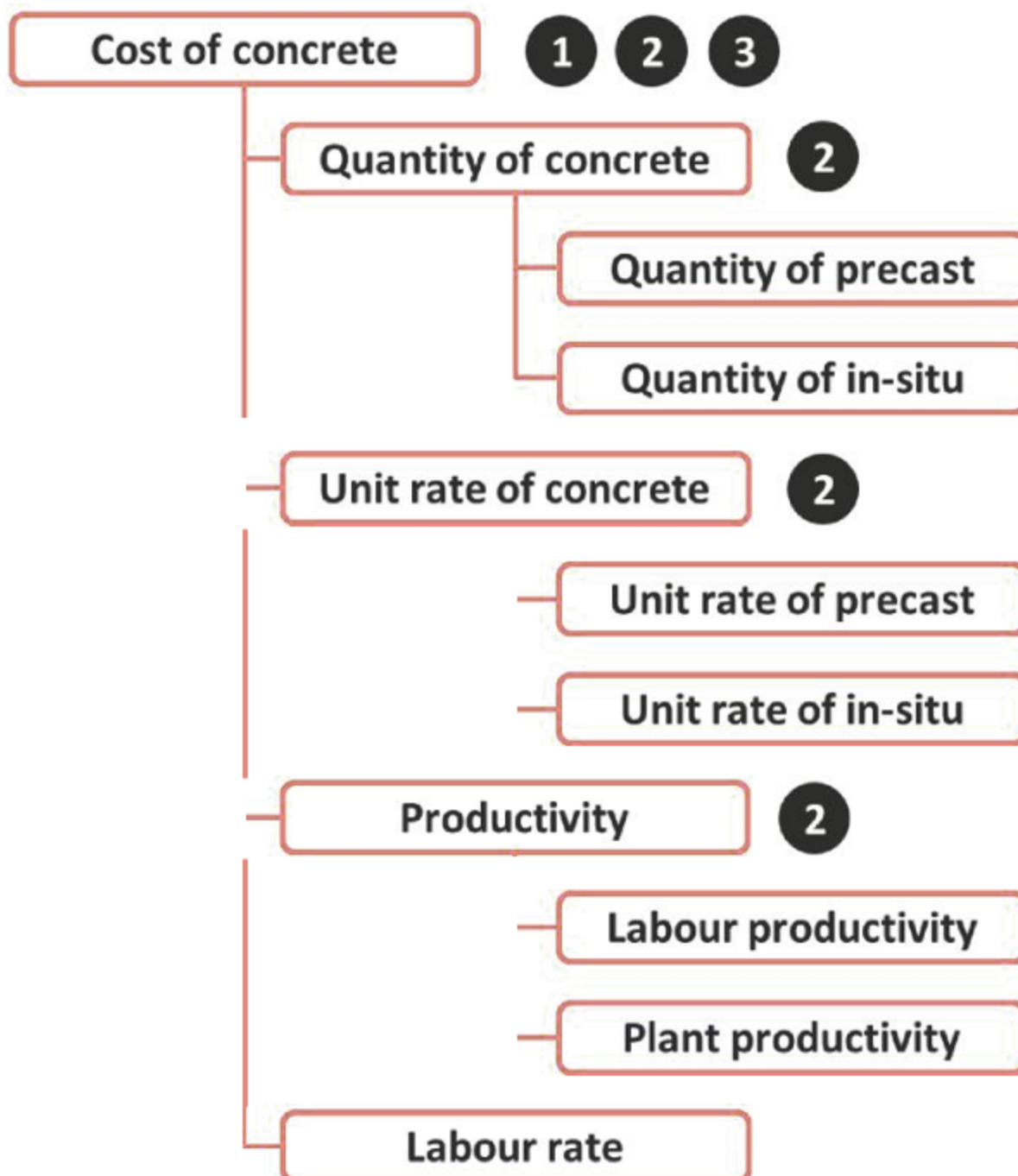


Table A 2: Deriving risk factors for concrete structures

Factor	Reason for breaking it down	
Cost of concrete	1	Uncertainty in bulk material quantities, unit rates, productivity and the labour rate are completely different from one another and will be assessed more reliably if they are addressed separately.
	2	There are different types of concrete (precast and in-situ) subject to differing levels of uncertainty and they must be separated to allow them to be treated appropriately in the model.

Factor	Reason for breaking it down	
	3	Labour cost and possibly productivity uncertainty that affect this cost will affect other costs in the same way as one another and need to be broken out to avoid building in complex correlations.
Quantity of concrete	2	There are different types of concrete (precast and in-situ) subject to differing levels of uncertainty and they must be separated to allow them to be treated appropriately in the model.
Unit rate of concrete	2	There are different types of concrete (precast and in-situ) subject to differing levels of uncertainty and they must be separated to allow them to be applied to the relevant costs in the model.
Productivity	2	There are different types of productivity uncertainty (labour and plant) subject to differing levels of uncertainty and they must be separated to allow them to be applied to the relevant costs in the model.

The uncertainty in the cost of concrete structures is not homogeneous. It consists of the combined effect of at least four sources of uncertainty and some of these, labour rate and productivity, may be relevant to other costs as well. Very often, each of those four items will be fairly homogeneous in themselves across a whole project or within one section of it. For example, the uncertainty around quantity estimates for concrete usually reflects the level of development of the design and this is usually fairly uniform across a project or within a major area of a project. However, if the structures consist of a mix of pre-cast concrete and concrete poured in-situ, and the uncertainty in the quantities and rates of these are different, the quantity factor and rate factor might need to be broken into two and applied to different parts of the cost for these two categories of concrete. It might also be that the labour productivity uncertainty arises from quite different matters to plant productivity uncertainty and it is easier to think about and assess the effect of each of these separately.

Temporary facilities costs often include some that are time dependent, such as the monthly cost of offices, vehicles, temporary fencing and on-site catering, and some that are essentially fixed, such as mobilisation and demobilisation. The part that is not time dependent will generally still be uncertain but its uncertainty will not be related to time. The part that is time dependent might suffer uncertainty about the running rate as well as uncertainty about how long the monthly cost will have to be paid. It is unusual to have to break down the monthly running rate or the non-time dependent costs into smaller parts to describe uncertainty. Project teams are usually happy to talk about the uncertainty in each of those as a whole unless, for example, there will be a large home office team as well as a large site team and their monthly running costs are subject to different levels of uncertainty.

Figure A 2: Deriving risk factors for temporary facilities costs

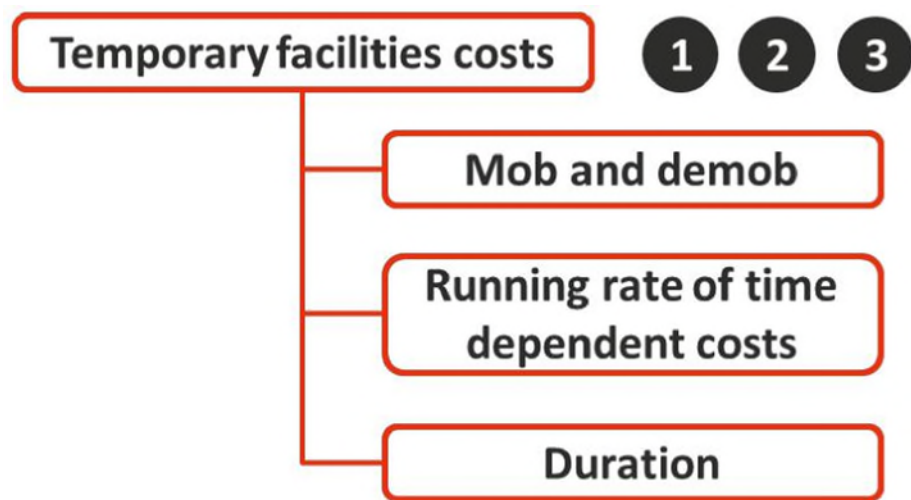


Table A 3: Deriving risk factors for temporary facilities costs

Factor		Reason for breaking it down
Cost of temporary facilities	1	Uncertainty in bulk mobilisation and demobilisation costs, the running rate of monthly costs and the duration of the project are completely different from one another and will be assessed more reliably if they are addressed separately.
	2	Time dependent and time independent costs are subject to completely different behaviour and, so long as mobilisation and demobilisation, which do not generally depend on the project's duration, are a significant part of the cost, they have to be separated to model them realistically.
	3	Project duration uncertainty will affect other costs, such as the home office and Owner's costs, in the same way and has to be broken out to avoid building in complex correlations such as where the running rate of the costs for the home office and Owner's team are subject to different levels of uncertainty and these both interact with the duration uncertainty.

In some cases, a direct assessment of the uncertainty in the duration of the work is sufficient to represent the effect of schedule uncertainty on the time dependent costs. In others, a separate schedule modelling exercise, which breaks down the schedule into major areas of work and describes the relationships between them, might be necessary.

# Appendix B – Simulation and statistical analysis

As a general rule, decision makers prefer lower estimates to higher ones. The reason being is that there may be a belief that if estimates are lower, either more projects can be developed within limited available funding, or proposed projects are more appealing to funding appropriators. However, this ignores the reality that the final project cost is independent of whether the initial cost estimate was arbitrarily high, low, or otherwise. A project for which the cost estimate is artificially low because risks have not been accounted for is likely to exceed its budget, whereas a project for which the cost estimate includes excessive contingency ties up funds that could be used for other projects. It is therefore paramount that the methodology used to quantify the risk is theoretically sound, but also enables cost estimators to build and interpret realistic models that provide decision-makers with clear and understandable results for budgetary purposes.

## Probability tools

Statistical modelling techniques include statistical simulation and statistical analysis. Although the goal is the same, the modelling techniques differ which will be discussed in more detail.

## Statistical simulation

Statistical simulation is a numerical experiment designed to provide statistical information about the properties of a model driven by random variables. The strong law of large numbers is the principle upon which simulation is built<sup>38</sup>. It says that the larger the sample size (i.e. the greater the number of iterations), the closer their distribution will be to the theoretical distribution (i.e. the exact distribution of the model's output if it could be mathematically derived).

The statistical simulation process follows these steps:

1. Define numerical experiment (spreadsheet, schedule network, etc.).
2. Define probability distribution functions (PDFs) for each random variable.
3. Define correlation coefficients for random variables.
4. Determine the number of experimental trials.
5. For each trial:
  - A. Draw correlated random variable(s) from defined PDFs
  - B. Compute the experimental results
  - C. Save the experimental results
6. At the end of the simulation, determine the statistics from the experimental results.

## Sampling techniques

Statistical simulation tools most commonly use either:

- Monte Carlo sampling: New sample points are generated without taking into account the previously generated sample points.
- Latin Hypercube sampling: Each variable is divided into  $m$  equally probable divisions and sampling is done without replacement for each set of  $m$  trials. Latin Hypercube takes roughly 30% fewer trials to achieve similar accuracy to basic Monte Carlo sampling.

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<sup>38</sup> Vose, D (2008) Risk Analysis: A Quantitative Guide, John Wiley and Sons, Ltd



Commercial Excel add-ins such as @Risk normally utilise Latin Hypercube sampling. Other approaches that can improve the efficiency of the sampling process further include Sobol sequences, Faure sequences and Niederreiter sequences<sup>39</sup>.

The realism of the simulation results depends on the reasonableness of the user inputs, correct modelling of PDFs for all random variables, and the correct specification of correlation between these PDFs (even if it assumed to be 0).

The advantages of statistical simulation are its ability to provide the statistics of a simulated PDF formed by complex mathematical modelling of random variables, and, its ease of use. Statistical simulation requires a certain discipline. Users should be aware of these constraints to ensure that their models have taken them into account.

## Statistical Analysis

Unlike simulation, statistical analysis relies on the exact calculation of moments of the PDF. Moments are properties of random variables. There are many of them. The moments of most relevance to statistical analysis are raw moments, central moments and standardised moments explained further as follows:

- The zeroth moment is the total probability (i.e. one). The first moments about the origin are called “raw moments”. The mean is the first raw moment of  $X$  about the origin, and is a measurement of the central tendency of the data.
- Central moments of a distribution are the raw moments about the mean. The first central moment is by definition zero, but the second central moment is the variance, which is a measure of dispersion about the mean.
- Finally, we have the standardised (or normalised) moments, the most well-known being skewness (the third standardised moment), and kurtosis (the fourth standardised moment).

Method of Moments is one example of an analytical technique used to calculate the moments of probability distributions. The method becomes very complicated if any of the component distributions are correlated.

While statistical analysis is a valid method of deriving the statistics of composite probability distributions, such as the distribution of total cost subject to the distributions of component costs (and thereby arrive at a probabilistic cost estimate), the Department recommends that a proprietary statistical simulation tool is used for the following reasons:

- Available simulation tools in most cases have had years of development and thus come with an implied level of reliability and sophistication through testing and upgrades.
- Simulation tools come with an abundant supply of ready to use charts and tables to analyse and report results. Those building custom models need to also develop the necessary charts and tables.
- Most analysts will find it difficult and inefficient to develop custom analytical models for every estimate. The use of simulation tools enables an analyst who is not an expert in the mathematics (of say, method of moments) to develop sophisticated, credible uncertainty analyses.
- Changes to a simulation model can be made very quickly and the results compared with previous models much more easily than with analytical techniques.

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<sup>39</sup> Mascaro S (2014) Making Robust Decisions with a Model Subject to Severe Uncertainty, Report Developed for the Department of Agriculture in conjunction with CEBRA (downloadable at [http://cebra.unimelb.edu.au/\\_\\_data/assets/pdf\\_file/0011/1378676/Final\\_Report\\_RRRA\\_Decisions\\_with\\_uncertain\\_models\\_2014-07-111.pdf](http://cebra.unimelb.edu.au/__data/assets/pdf_file/0011/1378676/Final_Report_RRRA_Decisions_with_uncertain_models_2014-07-111.pdf))

### Why run a simulation?

Simulation has long been used for analysing systems and decision problems. The Prussian army would simulate wars by holding field exercises in the woods of Europe in all kinds of conditions, a practice that continues throughout the world today. Simulation is used to forecast the weather and to train pilots. NASA uses simulation to predict rocket and satellite trajectories. Simulation can even be seen in games. Monopoly simulates the real estate market by using dice as a means to generate random events.

The problem with manual simulations such as moving troops through the field is that they are very time consuming. What's more, while physical exercises such as this can teach participants a great deal, one or two simulated outcomes provide very little information on which to base a decision about future events. However, if the simulations(s) can be implemented on a computer, thousands of replications can be performed in a few seconds, providing far more information to enable evaluation of the risks of projects and to help identify optimal solutions<sup>40</sup>.

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<sup>40</sup> Evans, J.R., and Olsen, D.L. (1998) Introduction to Simulation and Risk Analysis. Prentice Hall

## Appendix C – Correlation

The terms *dependency* and *correlation* are often used interchangeably but they have quite specific meanings. A dependency relationship in risk analysis modelling is where the sampled value from one variable (the *independent*) has a statistical relationship that affects the value that will be generated for the other variable (the *dependent*). The dependent variable may be completely determined by the independent variable or it might exhibit some uncertainty that is not due to the independent variable.

Correlation is a statistic used to describe the degree to which two variables are related. While dependency presumes a causal relationship between the variables, correlation need not<sup>4241</sup>.

There are three reasons correlation may be observed between data:

- There is a logical relationship between the two (or more) variables.
- There is another external or underlying factor affecting both variables.
- An apparent correlation has occurred purely by chance and no correlation actually exists.

Many costs in a project will be linked because there is a common cause or driver that affects each in a similar way. Usually this dependence, or correlation, will be positive; it is rare that an increase in costs in one area will be offset by corresponding benefits in another, one going down as another rises, because of a common underlying influence. Such offsets do happen by chance of course when two costs are uncorrelated so that one might rise as another falls but systematic relationships of this sort are unusual.

When performing a simulation, in most cases there are a potentially infinite number of possible combinations of scenarios that can be generated. Each of these scenarios must be potentially observable in real life. The model must, therefore, be prevented from producing, in any iteration, an event that could not possibly occur.

One of the restrictions that must be placed on a model is the recognition of any interdependencies between components. For example, if concrete pipe culvert components were to be sourced from the same supplier, the unit cost for supply of 450mm diameter pipe and 600mm diameter pipe would be expected to move in tandem. They are strongly positively correlated; if the price of one goes up or down, so will the other. If the interdependency of these two components is not modelled, the joint probabilities of the various combinations of these two parameters will be incorrect. Impossible combinations will also be generated, such as one cost rising while the other falls even though they are linked by a common source of uncertainty.

Such models will produce an output distribution that cannot be relied upon, or interpreted as, a reflection of real-world behaviour. It will contain impossible scenarios and thus provides misleading information about the reality that it is trying to represent.

For example, say an analyst has determined that the rate for a 450 mm reinforced concrete pipe (RCP) is realistically represented by a triangular distribution T(350,370,425) and the rate for a 600 mm RCP can be represented by T(410,430,495). It can easily be imagined from these inputs that, if these pipes are treated as independent variables (i.e. not correlated), in certain iterations the simulated cost for a 450 mm RCP will be greater than for a 600 mm RCP. As an example, the simulation software, in a particular iteration, may randomly select a rate for the 450mm RCP of \$422 and randomly select a rate of \$415 for the 600mm RCP. This is a situation that in all likelihood, would not occur in reality.

There are several ways of accounting for correlation in cost-risk analysis. Some of the main methods are:

- Using structural links, using an excel formula to reflect (or approximate), a direct functional relationship between quantities in a model, such as the cost of overheads and a project's duration.

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<sup>41</sup> Vose D (2008) Risk Analysis: A Quantitative Guide

- By aggregating inputs and assessing the risk together. This can be useful where there are a number of smaller elements which are similar in nature but for which it is difficult to define a direct relationship, such as where negotiations will influence labour for several different trades at a site and the uncertainty in their labour rates can be assessed as a whole.
- Using a correlation matrix for three or more values that are all related to one another, which accounts for the correlation by using a matrix of related inputs, with a correlation coefficient that defines the strength and sense (positive or negative) of their relationships. For example, a coefficient of 0.5 specifies that when the value sampled for one input is high, the value sampled for the second output will tend to, but not always, be high.
- Using a risk factor approach, as described throughout this guidance note, which disaggregates the sources of uncertainty. By defining the sources of uncertainty and determining their impact on the different cost elements, the correlation between cost elements is automatically dealt with because the relationships are implicitly included within the risk model.

## Magnitude of correlation impact

From a cost analysis perspective, the strength of correlations between a project's WBS element costs affects the total overall cost risk, as measured by the variance (from which the standard deviation is derived) of the total cost probability distribution. Ignoring any actual positive correlation between one or more pairs of WBS element costs can significantly understate a project's potential variation from its central tendency and so the "true" cost risk.

The statistical mean of a project's total cost is the sum of the statistical means of its WBS element costs. However, the statistical variance of the project's total cost is not the sum of the statistical variances of its WBS element cost unless the element costs are completely uncorrelated. To see why, note that the variance of the sum of two random variables  $X$  and  $Y$  is given by:

$$(X + Y)^2 = X^2 + Y^2 + 2XY$$

The variance of  $(X + Y)$  is not just the sum of the variance of  $X$  plus the variance of  $Y$ . The last term,  $(2XY)^2$ , is the co-variance between  $X$  and  $Y$ .  $\rho$  is the Pearson product-moment between  $X$  and  $Y$  and  $\sigma_X \sigma_Y$  is the product of their respective standard deviations. Thus, correlation between WBS items can have significant effects on the magnitude of cost risk, as measured by the standard deviation of the total cost probability distribution.

Suppose that we have a project with cost elements  $C_1, C_2, \dots, C_n$

If each  $C$  has a variance  $\sigma^2$  and the correlation ( $\rho$ ) between each  $(C_i, C_j)$ , which must be  $\geq -1$  and  $\leq 1$ , then the total variance  $(C) = n\sigma^2(1+(n-1)\rho)$ . The table below shows the variance when the correlation is 1, 0, or a known value,  $\rho$ .

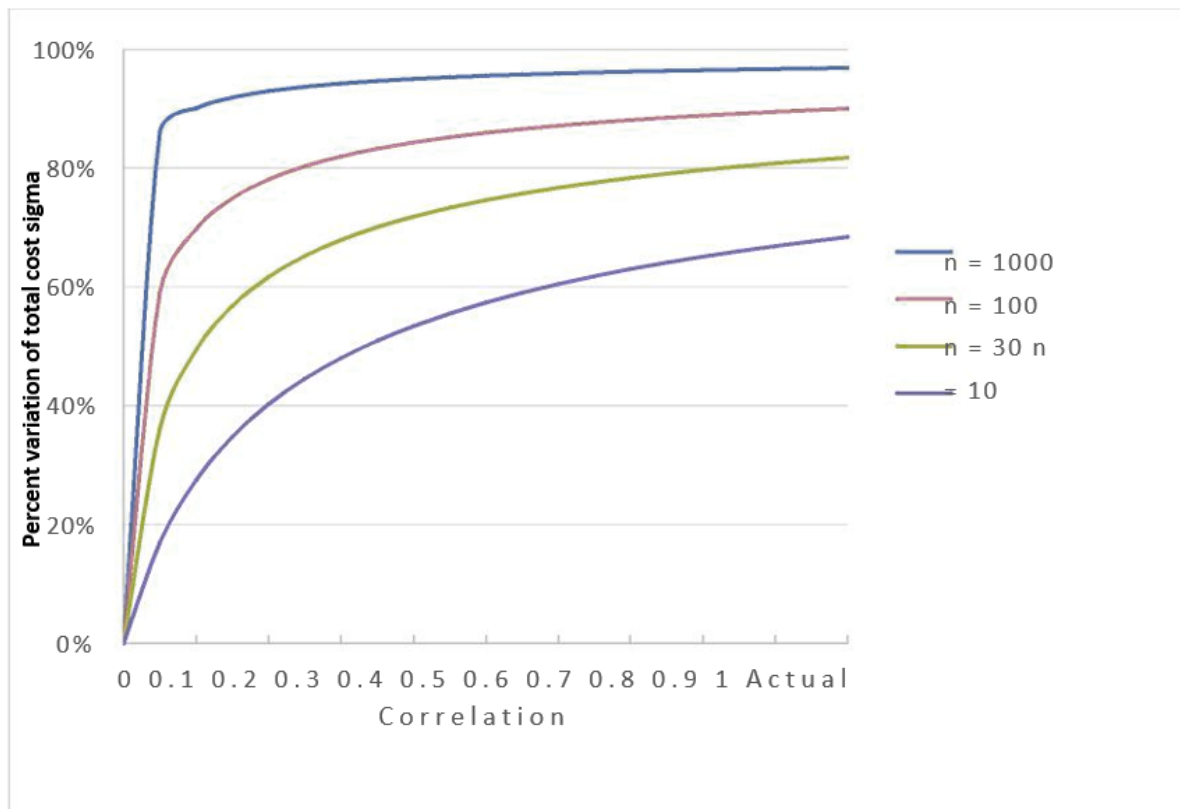
Correlation	0	$\rho$	1
$Var(C)$	$n\sigma^2$	$n\sigma^2(1+(n-1)\rho)$	$n^2\sigma^2$

When correlation is assumed to be 0 and its actual value is  $\rho > 0$ , the percent underestimation of Total-Cost Sigma (standard deviation) is 100% multiplied by:

$$1 - \sqrt{\frac{1}{1 + (n-1)\rho}}$$

where  $n$  = the number of inputs and  $p$  = the actual correlation<sup>42</sup>.  
This is demonstrated in the **Figure C1** below:

**Figure C 1: Maximum possible underestimation of total-cost sigma**



For example, suppose the correlation,  $p$  was assumed to be zero between all WBS elements in a 10 element WBS. If it later became evident that  $p$  was actually 0.5 between all WBS element costs, the maximum possible underestimation of total cost standard deviation is 57%. This does not mean that the risks have been underestimated by 57%; it is the total standard deviation that has been underestimated.

To understand how this relates to underestimation of risk, consider a system with 10 subsystems, each with a mean total cost equal to \$10 million and a standard deviation equal to \$3 million. If each of these subsystems in turn has 10 elements each, giving a total number of system elements of 100, assuming correlation is zero when it is actually 0.2 results in underestimating the P80 by approximately 8%. If correlation is actually 0.6, the P80 will be underestimated by approximately 15%<sup>43</sup>. note that unfortunately the impact cannot be derived from a simple formula as it depends on a number of inputs such as number of WBS elements, correlation coefficient selected, and choice and spread of distribution. Thus, the impact can only be found by running simulation trials.

A further example is shown below using a fictitious project which has been modelled twice.

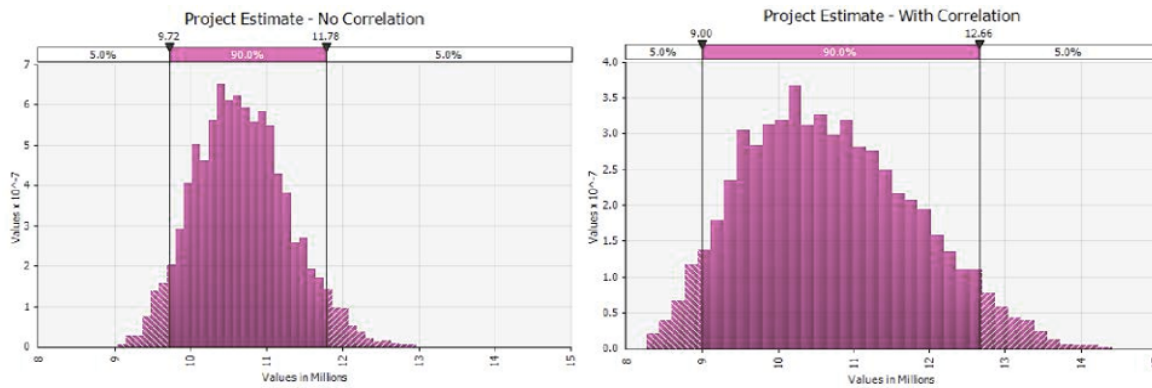
- with no correlation between cost elements.
- with full correlation between each cost element.

**Figure C2** shows the probably distribution function for both models, while **Figure C3** shows the respective cumulative distribution functions.

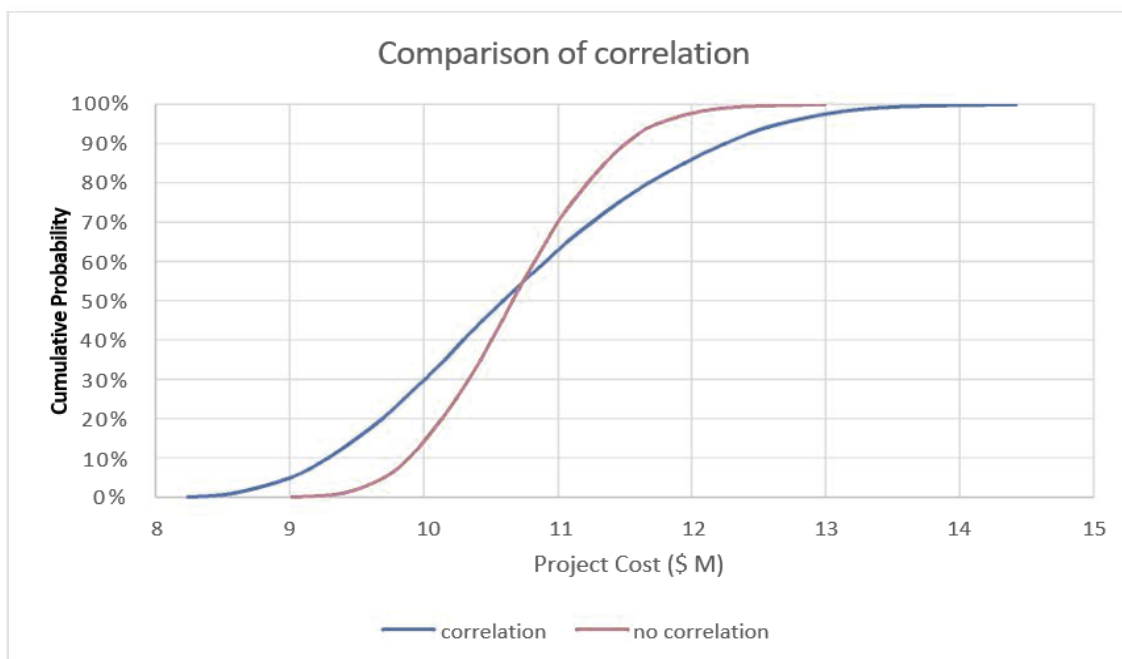
<sup>42</sup> Book, S, 1999, Why Correlation Matters in Cost Estimating

<sup>43</sup> Smart, C, Ph.D., CCEA,2013, Robust Default Correlation for Cost Risk Analysis

**Figure C 2: comparison of histograms without correlation (left) and with correlation (right)**



**Figure C 3: comparison of “S” curve of the same project with and without correlation**



Of note:

- When a project is modelled assuming cost elements are independent of each other, due to the central limit theorem (see **Appendix E** for a detailed explanation of the impact of the central limit theorem on output results), the output PDF closely approximates a normal distribution.
- Corollary to the above point and as expected, when correlation is accounted for, the final combined output PDF is more reflective of the input PDFs which were predominantly right-skewed in this particular example.
- Not accounting for correlation results in a steepening of the “S” curve and an associated loss of the extreme tails at either end by approximately 10%. In other words, if correlation is not accounted for, the model will not produce the minimum or maximum values that the project could actually take.
- Not accounting for correlation understates the absolute value of residual risk exposure beyond any chosen P-value. In this example the difference between the modelled project maximum and the P90 when correlation was accounted for is \$2.2 M; when correlation is not accounted for the difference is only \$1.4 M.

This example effectively demonstrates the balancing effect that occurs through not accounting for correlation between input variables. For example, if there is a +1 correlation between two variables, x and y, if the

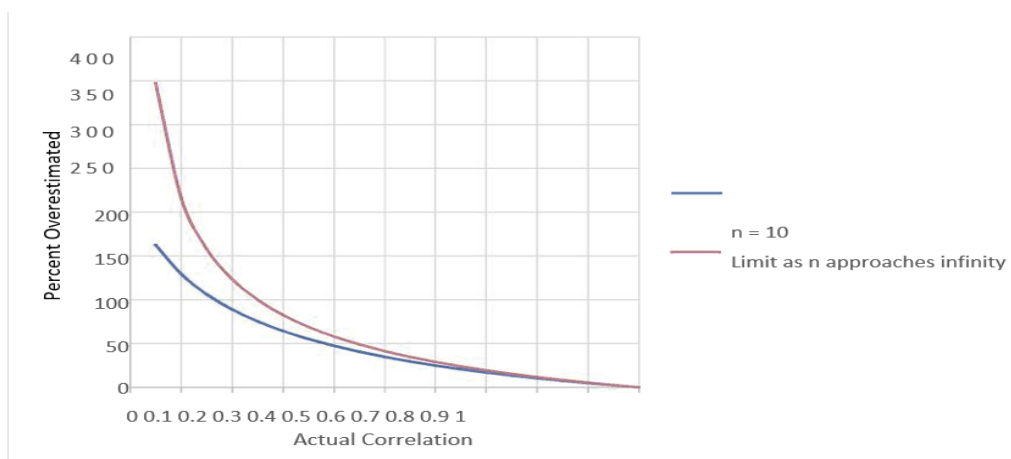
sampling routine picks a high value of x from its distribution, it will next pick a high value of y from its distribution. Strong correlations yield wider outcomes in outcomes because there is less chance of a high value in one area being cancelled out by a low value in another and vice versa<sup>44</sup>.

As well as being underestimated, the total standard deviation can also be overestimated if an excessively conservative coefficient of correlation is selected. When correlation is assumed to be 1 instead of its actual value, the percent overestimation of Total-Cost Sigma (standard deviation) is 100% multiplied by:

$$\sqrt{1 + (-1) - 1}$$

Figure C4 demonstrates the impact:

**Figure C 4: Maximum possible overestimation of total-cost sigma**



It can be seen that correlation affects variance, especially when summing large numbers of WBS

elements. Selecting appropriate correlation values between WBS elements is important to ensure that risk is neither under, nor overestimated.

## Strategies to account for dependencies when data is lacking

When dealing with correlations and dependencies in Monte Carlo simulation, it is important to bear in mind the potential complexity of dependencies and to recognise that empirical information is rarely available<sup>45</sup>. Just as many aspects of an estimate rest on the experience and expertise of skilled personnel, so does the assessment and modelling of dependencies and correlations.

Where there is reason to believe that increases or decreases in the cost of a certain WBS element are likely to cause a corresponding increase or decrease in the cost of another WBS element, it is necessary to identify and account for correlation within the risk model in order to provide a realistic picture of the total cost variance.

A number of methods have been proposed as potential solutions when correlation cannot be determined from statistical or empirical means or eliminated from a model by careful design. Some of these include:

- Undertaking a sensitivity analysis (recommended)
- Assume independence – this method is extremely easy but as per the preceding discussions, is wrong and may be severely misleading.

<sup>44</sup> Hollmann, J 2016 Project Risk Quantification: A Practitioner's Guide to Realistic Cost and Schedule Risk Management, Probabilistic Publishing, Gainesville, Florida

<sup>45</sup> Tucker W & Ferson S (2003) Probability Bounds Analysis in Environmental Risk Assessments, Applied Biomathematics (downloadable at <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.5.6926&rep=rep1&type=pdf>)

- Using various different default correlation coefficients<sup>46,47</sup>.
- Causal Guess Method.

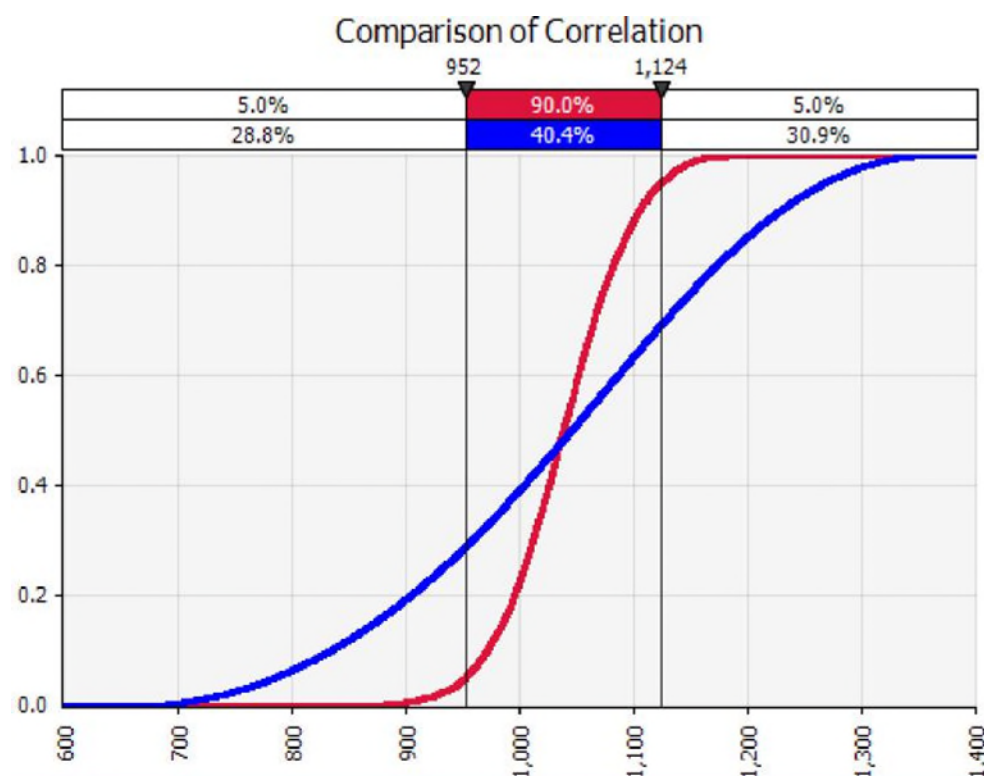
More sophisticated approaches can also be employed<sup>48</sup> however they are beyond the scope of this guidance note.

The preferred approach to account for correlation if not utilising a risk factor approach or robust methods such as aggregation or structural links, is to undertake a sensitivity analysis which allows a subjective assessment of the impact to be determined.

## Accounting for Correlation Using Sensitivity Analysis

The sensitivity of an analysis to correlation can be tested by first setting it to zero and then by setting it to 100% (i.e. using a correlation matrix with a correlation co-efficient of 1 applied to each model input) and then comparing results. An example is shown in **Figure C5** below where two “S” curves resulting from such an analysis have been overlayed on top of each other. Exactly the same model inputs have been used and run across two simultaneous simulations, the only difference being that correlation was set at zero in one simulation, and 100% in the other.

**Figure C 5: Comparison of correlation**



The gap, in terms of the dollar difference, between the two curves at the level of confidence chosen to fix the contingency indicates how significant an effect correlation has on the outcome. In the case of estimates for projects for which Commonwealth funding is being sought, the trigger points of P50 and P90 are the most relevant. In the example shown, there is negligible difference between the two curves at P50 and this will be

<sup>46</sup> Book, S, (1999), Why Correlation Matters in Cost Estimating

<sup>47</sup> Smart C (2013) Presentation to ICEAA, Robust Default Correlation for Cost Risk Analysis (downloadable at [http://cade.osd.mil/Files/CADE/References/101\\_2013\\_RobustDefaultCorrelation\\_Presentation\\_ICEAA\\_CSmart.pdf](http://cade.osd.mil/Files/CADE/References/101_2013_RobustDefaultCorrelation_Presentation_ICEAA_CSmart.pdf))

<sup>48</sup> Ibid



typical for most estimates. As such, it is the gap between the two curves at P90 that will generally be of most interest.

The example shows that there is a \$124m difference at P90. It also shows that when there is assumed to be no correlation, 90% of simulated values fall between \$952m and \$1,124m. When

there is assumed to be 100% correlation, only 40.4 % of values fall between \$952m and \$1,124m.

The procedure to produce such a comparison is as follows noting that for illustrative purposes @Risk<sup>49</sup> has been used in this instance:

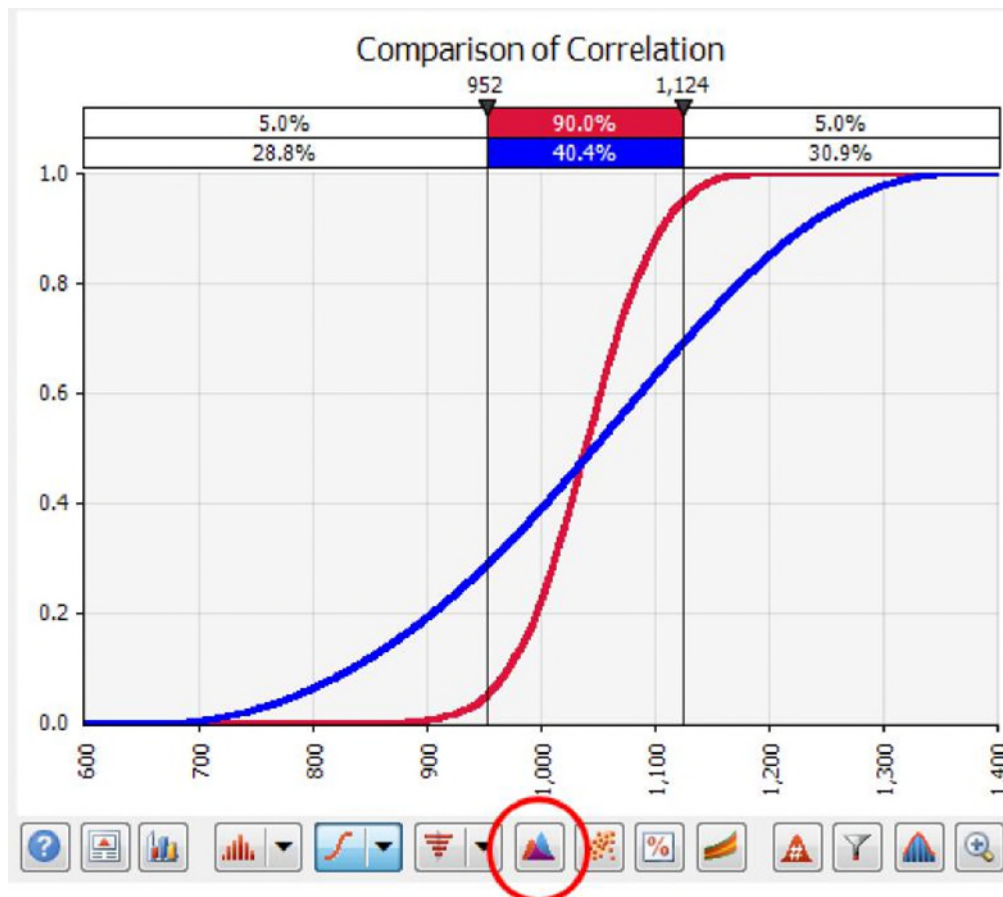
1. Begin either by creating a new model or by using an existing model. An example model with its basic inputs is shown below. The value in the “Simulated” column will change with each iteration of the simulation.

	A	B	C	D	E
1	Individual Risk Items	Optimistic (P10)	Most Likely	Pessimistic (P90)	Simulated
2	A	20	100	140	87
3	B	30	120	140	97
4	C	0	30	90	40
5	D	10	50	50	37
6	E	80	140	150	123
7	F	150	200	240	197
8	G	160	180	210	183
9	H	20	25	25	23
10	I	145	175	205	175
11	J	50	70	110	77
12					
13	Column Totals	665	1090	1360	1038

2. Create a correlation matrix which will correlate at coefficients of zero and one simultaneously in two simulations using the following steps:
  - a. Under define correlations from the ribbon, select define correlation matrix and choose a cell to locate the top left hand corner of the matrix within the worksheet.
  - b. Choose add inputs to select the cells with distributions to add to the matrix. In this example it is cells E2 to E11. The correlation matrix will be automatically created.
3. The coefficients must now be populated in the matrix. Select a cell outside the matrix and create the following function: =RiskSimtable({0,1}). This convention will ensure that in one imulation no correlation is applied, while in the second, simultaneous simulation, 100% correlation will be applied across all model inputs.
4. Each cell in the matrix should now be populated using an absolute cell reference to the function created in the preceding step. In the example shown each cell in the matrix = \$H\$14. It is a matter of but a few moments to drag the formula down and across to include each cell in the matrix. The model should look like the one below:

<sup>49</sup> The examples in this guidance note have been developed using the proprietary software programme @Risk and are used for demonstration purposes only. The Department does not endorse @Risk and acknowledges the availability of similar software tools.





## Determining an appropriate allowance for correlation

As discussed, the gap between the two curves at any confidence level shows how significant an effect correlation has on the outcome. If the gap is negligible, correlation can be ignored. If the gap is substantial, and 100% correlation is considered meaningful then the correlated version of the analysis can be used. If however, correlation is thought to exist, perhaps because several work packages will be affected by the same productivity uncertainties or commodity unit rates, but not at 100%, a subjective judgement will need to be made.

In the example, the difference between the two curves at P90 is \$124m. This difference should be examined, discussed, and documented by the project team in order to identify what could lead the estimate to end up at either end. This subjective assessment will be based on the experience of the project team, perhaps drawing upon examples of similar projects, and should also reflect the organisation's risk appetite. Once it has been determined and agreed what allowance for the impact of correlation is reasonable, this forms part of any decisions such as budgetary or funding allocations.

## Summary of correlation

Valid Monte Carlo specifically requires that the dependencies, or correlation, between the model inputs (the model line items) be defined, which includes being clear when there is no significant correlation. Correlations can have a large impact on the percentiles of the cost probability distribution and assuming no correlation can result in a large understatement of risk.

This appendix has briefly explored a number of techniques to quantify and account for correlation. However, the difference in approaches highlights how difficult it is to arrive at defensible assessments of correlation that are also technically sound. Methods that have arrived at suggested levels of correlation in one sector or type of work might not apply to a different type of project or even to the same nominal type of work when the technical, commercial or economic context changes.

The risk driver/risk factor modelling approach which is the focus of this guidance note describes cost uncertainty in a way that minimises the amount of correlation between model components that has to be modelled using correlation factors. It avoids the practical problems of dealing with a large number of interacting events or a large number of correlated line-item variations by focusing on the major estimating inputs that uncertainty will affect and which drive the individual line items.

The advantage of using risk factors to avoid building in the need to use correlation factors has been understood for decades<sup>50</sup>. As such, in order to overcome the inherent difficulties in dealing with correlation, the Department strongly encourages the use of a driver-based methodology wherever practicable.

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<sup>50</sup> Pouliquen L (1970) World Bank Occasional Papers Number Eleven

# Appendix D – Probability distribution functions

Shapes commonly used for routine cost modelling are the Beta PERT, Trigen, and triangular distributions. As demonstrated at [Section 6.4](#), the choice of shape for the individual distribution will normally have no material effect on the output. This is almost always the case, except where the extreme values of the output distribution are important.

Project funding decisions are generally not concerned with outcomes in the extreme tails of the distribution where different distributions may make a difference, but focus their attention between the 10th and 90th percentiles. The choice of distribution will rarely make a difference to the outcomes within this range.

Further explanation and detail regarding probability distribution functions is covered in this Appendix.

The realism of a risk analysis relies, in part, on the appropriate use of probability distributions to represent the uncertainty and variability of the system being analysed. The most basic property distinguishing the different of probability distributions offered by modelling tools is whether they are continuous or discrete.

## Continuous distributions

A continuous distribution is used to represent a variable that can take any value within a defined range (domain). For example, the height of a person picked at random has a continuous distribution. All values of height within the range found in a population are feasible.

## Discrete distributions

A discrete distribution may take any one of a set of identifiable values, each of which has a calculated probability of occurrence. The number of people in motor vehicles using a road is an example of a discrete distribution.

## Bounded and unbounded distributions

A distribution that is confined to lie between two determined values is said to be bounded. Examples are:

uniform – bounded between minimum and maximum.

- triangular – between minimum and maximum.
- beta – between 0 and 1.
- binomial – between 0 and a specified number, n.

Project variables that are strictly bounded include the number of days in a year when work will be affected by rain, between zero and three hundred and sixty five, or the proportion of cut material that may suitable for use as select fill, between zero and one hundred per cent. Many variables might not strictly be bounded but are so for all practical purposes, such as the length of a culvert at a creek crossing.

A distribution that is unbounded theoretically extends from minus infinity to plus infinity. A normal distribution is unbounded.

A distribution that is constrained at one end is said to be partially bounded. An example is the lognormal distribution which cannot go below zero but which has an upper bound of infinity.

Unbounded and partially unbounded distributions may need to be constrained to remove the tail of the distribution so that nonsensical values are avoided. For example, costs can exclusively never go below zero (otherwise they are not a “cost”) so if a cost element is modelled using a distribution that can generate a negative value it must be truncated. (While some contingent risks may present an opportunity for a saving over the base estimate, again there should generally be no chance that the cost can actually be below zero.)

The various Monte Carlo simulation software packages will allow for this functionality and will also allow a user to test for the theoretical minimum or maximum value that a modelled input can take

(RiskTheomin/RiskTheomax in @Risk for example) in order that the analyst can test whether truncating is required.

Analysts should be aware that that truncating the lower limit of a distribution moves the mean of

the distribution to the right, making it a more conservative estimate. This is less of a problem if the upper limit is also truncated.

## Choice of distribution

In principle, the correct distribution to assign is the maximum entropy distribution<sup>51</sup> which is the minimally prejudiced distribution that maximises the entropy subject to constraints supplied by available information. In simple terms, it is the distribution that is consistent with all the available information but includes no additional assumptions.

The difficulties in sorting out our true state of knowledge, and performing the intricate mathematics needed to find the maximum entropy distribution that represents it, makes attempting to define and assign the maximum entropy distribution highly impractical.

Various attempts have been made to identify the underlying empirical distribution of various sets of cost data, with publicly available information mainly related to Defence or space programs in the US. The 2010 US Air Force Cost Risk and Uncertainty Analysis Metrics Manual (CRUAMM)<sup>52</sup> identified the frequency of each distribution found across 1,400 fits of various cost data. A lognormal distribution was found to be the best fit to the data in 59% of instances followed by Beta (19%) and Triangular (18%). Uniform was never found to be, and Normal was rarely found to be the best fit.

It must be noted that these distributions were found to be the best fit to cost elements applicable to the US Air Force, predominantly communications devices, aircraft components and weapons systems, most of which are not directly comparable with Australian land transportation project cost elements.

In addition, observations of past projects are not necessarily a good guide to a new project at this level of detail. Systematic issues, such as discussed by Hollmann<sup>53</sup>, may be consistent within a sector under stable economic, technical and commercial conditions. While past history is a very valuable guide and can help us avoid unwarranted optimism, in an increasingly volatile world, we

cannot always assume that the lessons of the past will transfer directly to the work of the future. It is always necessary to consider the conditions of each new project.

There are a large number of possible distribution shapes defined in the literature and available through a variety of tools. In an effort to promote consistency across project estimates, unless robust statistical data is available suggesting an alternative, estimators are encouraged to limit their selection of continuous distribution to those defined in **Table<sup>54</sup> D1** below.

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<sup>51</sup> Cover, T & Thomas J (2006) Elements of Information Theory 2nd Edition

<sup>52</sup> Joint Agency Cost Schedule Risk and Uncertainty Handbook, March 2014

<sup>53</sup> Hollmann, J (2016) Project Risk Quantification: A Practitioner's Guide to Realistic Cost and Schedule Risk Management, Probabilistic Publishing, Gainesville, FL

<sup>54</sup> Adapted from AACE International (2008) RP No. 66R-11 Selecting Probability Distribution Functions for use in Cost and Schedule Risk Simulation Models

**Table D 1: Probability Distribution Functions for use in cost risk models**

PDF Name	Recommended parameters	Typical application	Knowledge of mode	Advantages	Disadvantages
Uniform Bounded	Lowest possible value, highest possible value.	For variables where a mode does not occur and/or the distribution shape is unknown.	No idea.	Simple to use. Can be easy for users to provide parameters.	Can overstate the probability of values in the extremes.
Triangle Bounded	Lowest possible value, mode, highest possible value.	Most likely value is clearly discerned and the shape of the distribution is not very highly skewed.	Good idea.	Easy for users to provide parameters. Its low central tendency can compensate for users that set low/high ranges that are too narrow.	Can overstate the probability of values on the skewed side of ranges when users set extreme low/high values.
Trigen Bounded	Same as triangle except the user defines what p-value the high and low values represent.	Same as triangle but used when it is more realistic to describe the spread of possible values in terms of say absolute minimum and maximum values is P10 and P90 because assessing	Good idea.	Provides a simple way to deal with team/expert input.	
		absolute minimum and maximum values is			



PDF Name	Recommended parameters	Typical application	Knowledge of mode	Advantages	Disadvantages
Normal Unbounded	Mean, Standard deviation.	Values that have symmetrical distributions and there is some empirical basis to define parameters.	In this case the mean, median and mode are equal to each other.	Works well for items with unskewed ranges. It can legitimately be used to represent the net effect of a set of uncorrelated values that are added up, such as sequential lengths of rail alignment	Difficult for users to objectively express a standard deviation. Most cost estimates are skewed. Can result in inappropriate negative values on the low end.
Lognormal Bounded on the low side at zero	Mean, Standard deviation of the variable's natural logarithm.	Values that have asymmetrical distributions and there is some empirical basis to define parameters.	Mean or median is known better than the mode.	Works well for items with either skewed or unskewed ranges.	Difficult for users to objectively express a standard deviation. The mode arising from a given mean and SD might not accord with the assessors' judgement
PERT or BetaPERT Bounded	Lowest possible value, mode, highest possible value.	Used where a most likely value is clearly discerned and the shape of the distribution is not very highly skewed.	Very good idea.	Can be easy for users to provide parameters. Less overweighting of extremes than Triangle.	If the team/expert exhibits optimism bias, may underweight the skewed sides.

The table above indicates the information that the estimator requires in order to select a distribution. It might be reasonably argued that since a triangular distribution rarely represents the underlying distribution of a cost element "in the real world", it is inappropriate to select it for modelling purposes. However, it lends itself well to road and rail projects because for most of the cost elements there will realistically be a most likely (mode) value, for example volume of asphalt and aggregates, number of pits and pipes required, etc., as well as a reasonably realistic minimum and maximum value. The triangular shape adds no assumptions to the analysis beyond:

- There is a peak.



- The cumulative probability outside the upper and lower parameters (either zero or a defined percentage).

Beyond this, the triangular shape adopts the simplest form that satisfies the three parameters used to define it, a straight line decline from the peak to zero.

While a lognormal distribution may at times be more representative of highly skewed values, such as the extent of an environmental incident with a high chance of a minor impact and a rapidly declining chance of increasingly extreme impacts, it requires an analyst have a better idea of the mean or median than the mode because of the way most modelling tools require it to be specified. Unless there is good reason to be certain that a distribution has a shape where the tails fall away quickly, as does a PERT distribution, the triangular form may be preferred. The tails of the triangular distribution are also given more weight than the tails of a PERT or similar shape and so do not introduce any bias towards certainty or false precision<sup>55</sup>. If anything, a triangular shape provides a conservative view of the potential spread of an uncertain quantity as it lends a little extra weight to the tails of the distribution.

There is a useful variation of the triangular distribution called Trigen in @Risk and other tools. The Trigen distribution requires five parameters:

- A low value.
- The most likely.
- A high value.
- The probability that the parameter value could be below the low value
- The probability that the parameter value could be above the high value.

The Trigen distribution is a useful way of avoiding asking experts for their estimate of the absolute minimum and maximum of a parameter. These are questions that experts often have difficulty answering meaningfully since there may be no theoretical minimum or maximum even though the likelihood of realistic outcomes becomes minute a short way beyond the low and high values. Instead the analyst can discuss what probabilities the experts would use to define low and high values. In this way the Trigen allows a modeller to elicit assessments in terms that an expert can respond to meaningfully and use those assessments directly in the analysis without further adjustment.

Analysts should be aware of the limitations of particular distributions when making a selection. As mentioned, project funding decisions usually focus their attention between the 10th and 90th percentiles, not the tails of the distribution. The choice of distribution will rarely make a difference to the outcomes within this range. However, for some kinds of analysis, such as safety, critical failure risk or assessment of contractual liabilities for commercial purposes, the tails may be the area of interest, and different approaches to analysis and more attention to the distribution shapes may be required. When this is necessary, a sensitivity approach is usually recommended, examining alternative distribution shapes and forming a judgement on the implications for a project, since there is no basis on which a specific shape can be selected.

## Distributions to model project specific risk

Probability distribution functions can be categorised as either parametric, or non-parametric. Most distributions – normal, lognormal, beta, etc - are parametric and are based on a mathematical function whose shape and range is determined by one or more distribution parameters.

Non-parametric distributions such as triangular, uniform, and discrete have their shape and range determined by their parameters directly and are generally more reliable and flexible for modelling expert opinion about a

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<sup>55</sup> Cooper D, et al, 2014 Project Risk Management Guidelines: Managing Risk with ISO 31000 and IEC 62198 2nd Edition

model parameter. An exception is the PERT<sup>56</sup> distribution, which although strictly a parametric distribution, has been adapted so that the expert need only provide estimates for the minimum, most likely, and maximum values, and the PERT function finds a shape that fits these inputs.

Similar to when selecting a distribution for a risk factor, the triangular distribution's appeal is that it is easy to think about the three defining parameters (minimum or low, most likely, maximum or high), and to imagine the effect of any changes.

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<sup>56</sup> The origins and limitations of the PERT form of the Beta distribution are investigated in the discussion paper Beta PERT Origins (Broadleaf 2014) available at: <http://broadleaf.com.au/resource-material/beta-pert-origins/>

## Appendix E – Number of line items to model

The greater the number of independent (i.e. uncorrelated) inputs that are put into a model, the tighter the distribution will become as it converges to a normal distribution and the extreme tails of heavily-skewed marginal distributions are lost.

Models with a very large number of line items are still valid, provided that the dependency between line items is accounted for, noting that, in general, there will be various dependency relationships between cost elements on infrastructure projects. If this is not done then the greater the number of (uncorrelated) cost elements included in the model, the less representative of reality it becomes, particularly at higher P-levels, and contingency is likely to be underestimated.

Probabilistic models should be limited to between 20 and 40 inputs, (preferably 20 or less) in order to maintain the validity of the results.

This appendix provides further explanation of the importance of limiting the number of inputs into a model.

The central limit theorem (CLT) says that the mean of a set of  $n$  variables (where  $n$  is large), drawn independently from the same distribution will be normally distributed. That is, the average of the sum of a large number of independent, random variables with finite means and variances converges to a normal random variable.

This theorem also applies to the sum (or average) of a large number of independent variables that have different probability distribution types, in that their sum will be approximately normally distributed providing no variable dominates the uncertainty of the sum.

How large  $n$  has to be depends on the individual distributions, but in practice the convergence to the Gaussian, or normal, distribution is surprisingly fast.

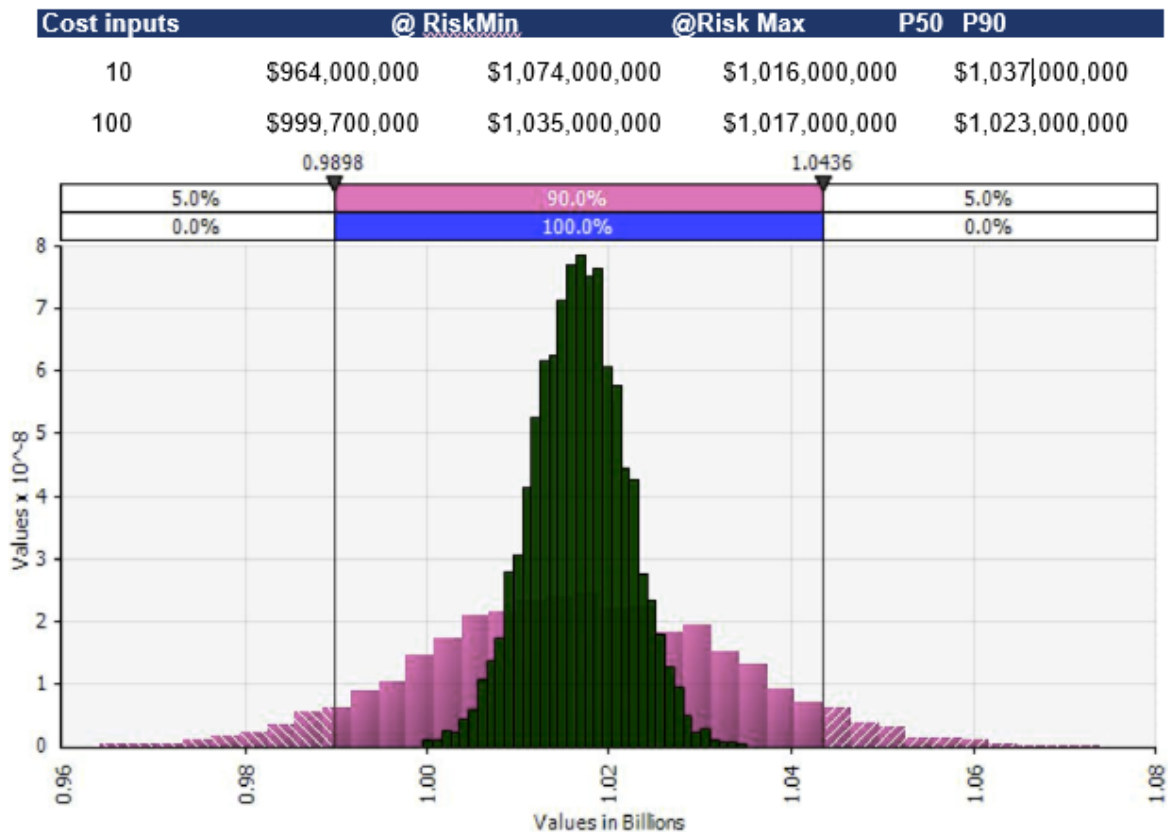
Because most risk analysis models are a combination of adding and multiplying variables together, it should come as no surprise that most risk analysis results appear to fall between a normal and lognormal distribution. A lognormal distribution also looks like a normal distribution when its mean is much larger than its standard deviation, so a risk analysis model result even more frequently looks approximately like a normal distribution, being a roughly symmetrical bell shaped curve.

Leading from the discussion of the CLT above, it is apparent that the greater the number of independent (i.e. uncorrelated) inputs that are put into a model, the tighter the distribution must become as it converges to the Gaussian and the extreme tails of heavily-skewed distributions are lost. This is best illustrated by a simple example. Consider a project with 10 independent cost elements, each with a most likely value of \$100 million with an uncertainty represented by a triangular distribution with a low of -10% and a high of +15%.

Now consider a case of 100 independent cost elements with the same distribution but with a most likely value of \$10 million so that the total base estimate remains the same.

Two simultaneous simulations of 5,000 iterations based on the inputs as described above gives the following results:

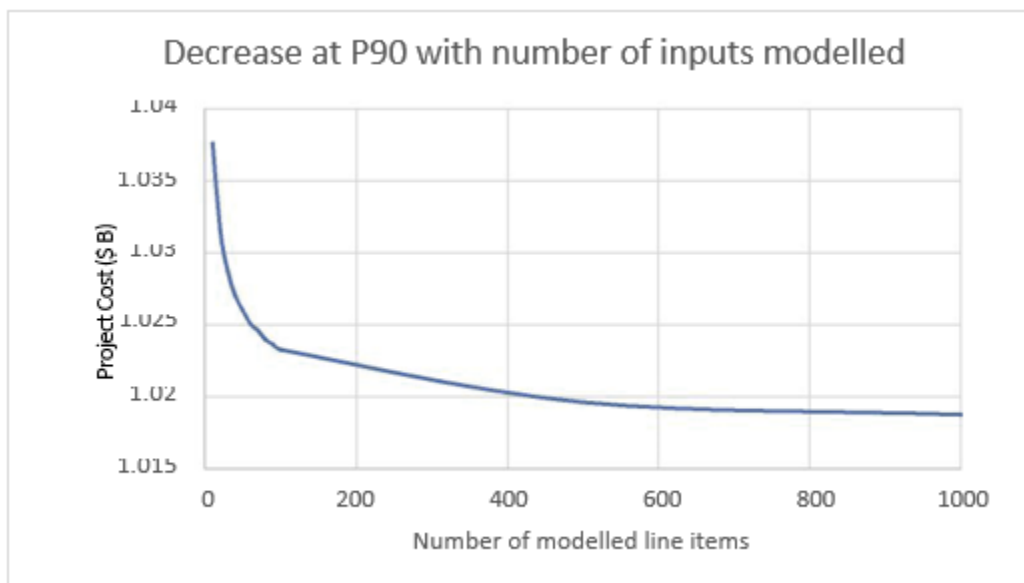
**Figure E 1: 10-input results overlain by 100-input results**



It can be observed that increasing the number of inputs reduces the tails (both minimum and maximum) of the distribution significantly. As expected the P50 essentially remains the same, however the P90 is substantially reduced.

Extending the above analysis further, **Figure E2** plots the change in P90 as the number of inputs increases.

**Figure E 2: Decrease in project cost( P90) with increasing number of inputs**



The results above suggest a somewhat linear relationship to describe the decrease at P90 as the number of modelled line items increase up to about 30 items. By 100 items, most of the tails are lost. Beyond this

number the effect is less marked, because as can be seen in the table, both the P90 and maximum asymptotically approach the P50.

The results above suggesting that the model inputs should be limited in number is consistent with findings<sup>57</sup> that in virtually all projects estimates, the uncertainty is concentrated in a select number of critical items – typically 20 or less.

Increasing the number of line items can be thought of as analogous to the portfolio effect where the extent to which variations in returns on a portion of assets held are partially cancelled by variations in returns on other assets held in the same portfolio (see boxed text at the end of this appendix). As shown in the table above, this effect is most marked in the extreme tails. This is not necessarily a problem provided that the model is realistic and is the reason that the required contingency allowance, when aggregated across a project as a whole, will be less than the sum of contingencies allocated separately to individual project cost elements at the same percentile value.

A model with many components that should be correlated, but are not, will generally understate the contingency required to provide confidence in the adequacy of funding so this form of modelling error creates risk for project owners and managers.

Consistent with [Section 2](#), disaggregating the sources of uncertainty rather than the costs themselves, which are subject to multiple sources of uncertainty, is a more rigorous method of both dealing with correlation while retaining the clarity of judgement that results from an appropriate level of disaggregation. In most cases it will also result in a simpler, cleaner risk model with far fewer inputs than a line-item model while still retaining enough detail to represent the uncertainties affecting the cost.

### Contingency and variance

In 1952 an unknown graduate student at the University of Chicago, Harry Markowitz published a fourteen-page article titled “Portfolio Selection”. His aim was to construct portfolios for investors who consider expected return a desirable thing and variance of return an undesirable thing, in other words a strategy where variance of return is minimised. This is typically desirable for investors because the greater the variance or standard deviation around the average, the less the average return will signify about what the outcome is likely to be.

The impact of diversification was Markowitz’s key insight. A system with only a few strongly interacting parts will be unpredictable. Just like the example on page 34, while the expected return looks great, you may win or lose heavily with just one wager and the outcome is uncertain. Diversification reduces volatility because some assets while rise in price while others are falling, or at very least, the rates of return among the assets will differ.

The return on a diversified portfolio will be equal to the average of the rates of return on its individual holdings (recall that the means of individual distributions are additive), however its volatility will be less than the average volatility of its individual holdings. Diversification means you can combine a group of risky securities with high expected returns into a relatively low-risk portfolio with one caveat - the covariances, or correlations, among the returns of the individual securities must be minimised.

Markowitz’s paper formed the basis for much of the theoretical work in finance that followed and highlighted the need to consider risk (or more specifically variance as a proxy for risk) rather than just expected return. While methods today are more sophisticated, the idea that the value of the total project risk is less than the sum of the project’s individual risks is built on much the same premise.

Markowitz, H., (1952) Portfolio Selection, The Journal of Finance, Vol 7, No.1. pp. 77-91

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<sup>57</sup> AACE International (2008) RP No. 41R-08 Risk Analysis and Contingency Determination Using Range Estimating, AACE International, Morgantown, WV

# Appendix F – Number of iterations to run in a simulation

The number of iterations required for convergence is dependent upon the model being simulated and the distribution functions included in the model. For example, more complex models with highly skewed distributions might require more iterations than simpler models, however 5,000–10,000 iterations are likely to be adequate for the majority of land transportation projects.

This appendix explores certain concepts, including statistical error, in greater detail which suggest that 5,000 – 10,000 iterations are appropriate in the majority of cases.

In determining how many iterations to run in a simulation, there are generally two opposing pressures:

- Too few iterations will fail to sample a representative number of outcomes, often indicated by output histograms that are lumpy, and result in inaccurate conclusions being drawn from the outputs.
- Too many iterations may take a long time to simulate, a long time to produce graphs, and there may be limitations (row limitations, etc.) if exporting data to Excel.

The law of large numbers concerns the behaviour of sums of large numbers of random variables. It can essentially be interpreted as a statement that a Monte Carlo estimate converges to the correct answer as the random sample size approaches infinity<sup>58</sup>. For the purposes of this guidance note it is sufficient to say that the above can be proven<sup>59</sup>.

The strong law of large numbers can be extended in a number of ways including application of the Central Limit Theorem, the Berry-Esseen Theorem, or the Bikelis Theorem in order to determine

the number of simulations to run to ensure convergence to a confidence interval on the accuracy<sup>60</sup>.

In the absence of any specified convergence criteria, checking for convergence can be done manually by undertaking the following steps<sup>61</sup>:

1. Set the number of iterations to at least 5,000 and run the simulation.
2. Record the statistics for:
  - Mean
  - Standard deviation
  - 5th percentile
  - Median (50th percentile)
  - 95th percentile
3. Perform additional simulations by increasing the number of iterations by increments of at least 1,000.
4. Monitor the change in the above statistics.

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<sup>58</sup> This interpretation is sufficient for practical purposes but not quite strictly correct. The law tells us that the average of a large sample will be more likely than the average of a small sample to vary from the true average by some stated amount. The key word is vary. What we are seeking is not the true underlying cost distribution, but for the probability that the error between the observed cost distribution and the true cost distribution is less than some stated value. note that Jacob Bernoulli explicitly excludes the case that there will be no error even if the sample size is infinite. See Bernstein pg 122.

<sup>59</sup> Calculations using standard properties of binomial coefficients can be used to prove the law of large numbers. Stewart, I., (2012) In Pursuit of the Unknown: 17 Equations that Changed the World. Basic Books, Great Britain

<sup>60</sup> Lapeyre B. (2007) Introduction to Monte-Carlo Methods (downloadable at <http://cermics.enpc.fr/~bl/Halmstad/monte-carlo/lecture-1.pdf>)

<sup>61</sup> NASA (2011) Probabilistic Risk Assessment Procedures Guide for NASA Managers and Practitioners 2nd Edition (NASA/SP-2011-3421) NASA

5. Stop if the average change for each statistic (in two consecutive simulations) is less than a desired value (typically in the range of 1% to 5%).

Alternatively, it is arguably simpler and more practical to use the in-built functionality of simulation software to check that the desired convergence has occurred.

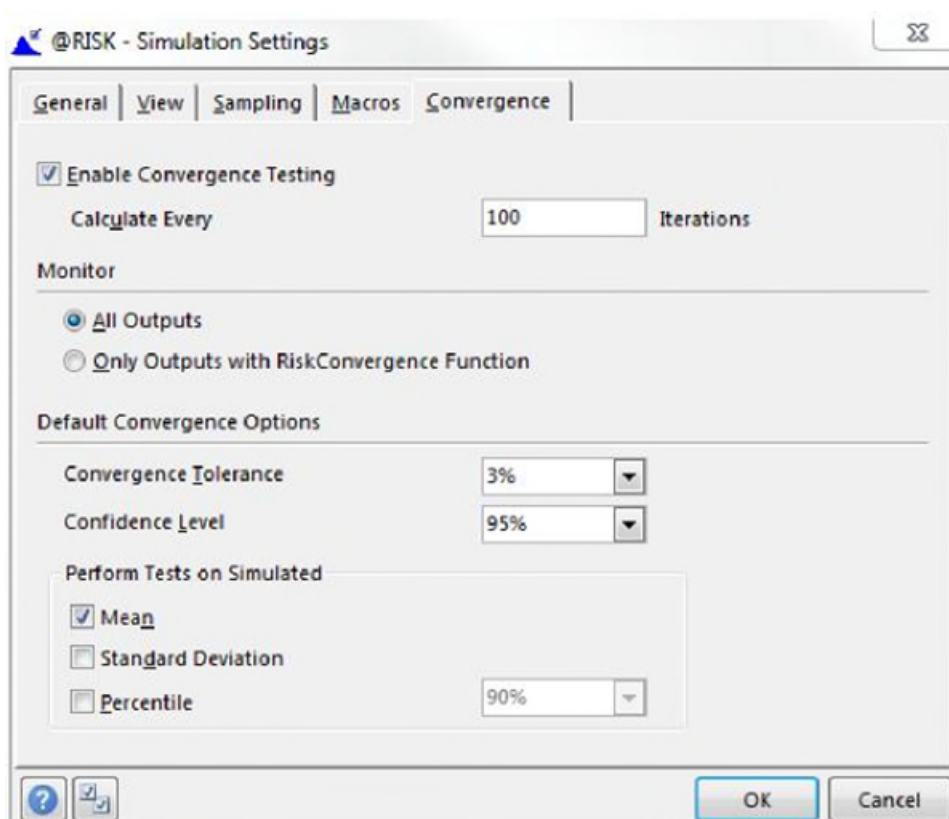
For example, after a simulation has been run on a \$10m project, one may wish to know with 95% confidence that the P90 has converged to within \$50,000 of the 'true' P90 (noting that any of these parameters can easily be adjusted if desired). On the same project, another analyst may be satisfied on a 95% confidence interval that the P80 has converged to within \$100,000 of the 'true' P80.

Rather than attempting to apply one of the various theorems, or record statistics manually from simulations, specifying the actual dollar amounts in this way gives greater meaning and provides a specific answer to the question: "Has the simulation converged?"

Convergence monitoring and testing can be undertaken by selecting the simulation settings in

@Risk<sup>62</sup> where the convergence tolerance and confidence level may be adjusted from their default levels as shown in **Figure F1** below. Some other tools offer similar facilities.

**Figure F 1: Adjusting convergence settings on a simulation. Source: @Risk User's Guide Version 7 August 2015, Palisade Corporation**



If there are any discrete risks, those that have a defined probability of causing one type of impact or one minus that probability of causing a disjoint impact, such as a 90% chance of zero impact and a 10% chance of

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<sup>62</sup> The examples in this guidance note have been developed using the proprietary software programme @Risk and are used for demonstration purposes only. The Department does not endorse @Risk and acknowledges the availability of similar software tools.



an impact lying between \$100m and \$120m with a most likely value of \$105m, the number of iteration might need to be larger than when there are only continuous distributions in a model. If the probability of such an event is very low, say 1%, only a very small number of iterations will include that event. If that event is important and its effect is to be included in the contingency, it may be necessary to run the model for a very large number of iterations to get a realistic view of its overall effect. However, unless the extreme tails of the distribution are important, such low risk events will rarely be included in the analysis and, for most routine project contingency management purposes, rare events of this scale are best handled as separate items. They might require analysis in their own right but there is no need to blend them in with the general contingency. In fact incorporating rare events into the general contingency is not recommended.

In summary, the number of iterations required for convergence is dependent upon the model being simulated and the distribution functions included in the model. For example, more complex models with highly skewed distributions might require more iterations than simpler models, however 5,000-10,000 iterations are likely to be adequate for the majority of land transportation projects.

It should be noted that the rate of convergence (at 95% confidence) is approximately  $\frac{0.98}{\sqrt{n}}$  so adding

one significant figure of accuracy requires increasing by a factor of 100. Attempting to achieve a high level of accuracy (i.e. a large number of significant figures) is unlikely to be practicable and is rarely necessary.

### Statistical error<sup>63</sup>

As defined by the formula in the preceding section, there is a strict mathematical relationship between sample size and statistical error. The larger the sample, the smaller the statistical error, or the more confident an analyst can be that a simulation has converged to the “true” underlying distribution defined by the model. Statistical error is a verifiable source of error in any simulation, and it is a consequence of randomness and a function of the size of the sample.

@Risk’s default convergence options are a Convergence Tolerance of 3% with a 95% confidence level, and these bear further explanation by way of an example.

Say a simulation indicates that the cost of a particular project at some defined percentile level is \$100 million. The analyst knows that statistical error (pure randomness) might affect the result of the simulation; the real cost might not be \$100 million, but could be \$99 million, or \$102 million, or even \$200 million if there was an extreme random event in the sample.

If an analyst says that the convergence tolerance (which may more correctly be termed the margin of error) is 3%, this is an expression of a defined level of confidence that randomness can only distort the answer by three percentage points up or down – the real cost is somewhere between \$97 million and \$103 million. However, this confidence isn’t absolute. By convention<sup>64</sup>, margin of error is a number larger than the imprecision caused by randomness 95 percent of the time (hence @Risk’s default confidence level being set at 95%). This means that on occasion an unlikely set of events could produce a result more than 3% away from the limit to which it would converge with an infinite number of iterations. Most of the time though – in 19 out of every 20 simulations, the results of the simulation will fall within 3% of that limit.

Note that a margin of error of 3% is achieved, in this case, through a simulation of approximately 1,000 iterations. The margin of error (to two decimal places) for 5,000 and 10,000 iterations is shown below<sup>65</sup>:

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<sup>63</sup> This section adapted from Seife, C (2010) Proofiness: How you are being fooled by the numbers, Chapter 4

<sup>64</sup> It was English statistician and biologist R. A. Fisher who proposed a 1 in 20 chance of being exceeded by chance as a limit for statistical significance. He also developed the analysis of variance (ANOVA) and was one of the founders of population genetics. ([https://en.wikipedia.org/wiki/Ronald\\_Fisher](https://en.wikipedia.org/wiki/Ronald_Fisher))

<sup>65</sup> Abraham de Moivre was an early contributor to the modern theory of probability. He wasn’t satisfied with the Law of Large Numbers which says, for example, that the proportion of heads in a sequence of flips gets closer and closer to 50%; he wanted to know how much closer. It was his insight that the size of the typical distribution discrepancy is governed by the square root of the sample size. See Ellenberg



- For 5,000 iterations, the margin of error =  $0.98 / \sqrt{5000} = 1.39\%$
- For 10,000 iterations, the margin of error =  $0.98 / \sqrt{10000} = 0.98\%$

Thus it should be borne in mind that when using Monte Carlo simulation, the final outcome is an approximation of the correct value with respective error bounds, and the correct value is within those error bounds.

Margin of error is a somewhat subtle concept but can be considered as no more than an expression of how large the sample is. While this may suggest that running a greater number of iterations in a Monte Carlo simulation will give more accurate results, this thinking is highly misleading. The margin of error only represents statistical error and while this error is important, there are far more potential errors which can affect the estimate of a project arising from a simulation to a much greater extent. When a project estimate is later proven to have been spectacularly wrong, the problem will almost never have been caused by statistical error. It is more likely to have been caused by the inputs going into the quantitative model itself being highly flawed – inappropriate ranges and/or distributions, not accounting for correlation, misunderstanding or non-identification of major risk factors, and other systematic errors.

When an analyst uses convergence as the sole test of whether to believe the results of a simulation, they are blind to the sources of error that are most likely to render the results of the simulation meaningless.

It is also worth noting that the precision that can be attributed to the subjective inputs of a model is limited and will often render the possible variation associated with convergence irrelevant. This is not to say that the subjective and uncertain nature of modelling inputs undermines the value of such analyses. It simply highlights that Monte Carlo simulation modelling of projects using assessments created by experienced personnel is not the same as using Monte Carlo simulation for the analysis of physical and engineering systems, for instance. It is a way of making sense and deriving value from the judgement of experienced people in the context of complicated interactions between design outputs, market information and implementation forecasts.

# Appendix G – Contingency allocation

A useful form of sensitivity analysis that can be applied to risk models is contingency sensitivity. This is an examination of what can be said to contribute most to the offset between the base estimate and the funding level corresponding to the required level of confidence for the project. This has to be interpreted with care as the entire foundation of a contingency is that variations in one part of the work will be balanced by variations in other parts (refer to Contingency and Variance within **Appendix E**). However, so long as it is not misused, it is a useful diagnostic tool and can be used to allocate contingency between work packages or stakeholders on an equitable basis, if this is necessary.

A common practice is to regard the contingency for an entire project as being distributed pro rata across all work packages. On this basis, if the overall contingency is 15% of the total base estimate, each work package would have a notional 15% contingency. This practice is fundamentally flawed.

If one work package is relatively predictable, a 15% contingency might be excessive and offer the work package manager a very low risk task. Another work package might house most of the uncertainty on the project and find a 15% contingency leaves it seriously exposed.

The preferred means of allocating contingency between work packages or stakeholders is on the basis of equitable provisions that represent the same percentile value in each area, leaving them all with the same level of risk or exposure within their own budgets. Because percentile values do not add to the same percentile of the total cost, it is necessary to search for a percentile value that, when applied equally to the component costs generates component contingencies that add to the value of the overall contingency. This is illustrated in <Risk factor model 1> in the contingency analysis tab.

The contingency analysis sheet in <Risk factor model 1> is set out as follows.

- The base estimate and simulated values for the lines in the estimate summary are read into columns B and C so these values are live and vary during the simulation.
- The variation between these two is calculated in column D.
- The mean value of the variations is calculated in column E, using the @Risk formula.
- RiskMean(). This is just for interest and is not used in what is discussed below.
- Percentile values of the variations are calculated in column F using the @Risk formula RiskPercentile().
- At the top of column F, the preferred percentile point at which to set the overall contingency is set in cell F1 and the percentile point at which to allocate the contingency to line items is set in cell F2.
- The value in F1 is used in cell D23 to calculate a contingency for the whole project.
- The value in F2 is used to derive percentile values for the individual line items.
- Cell F3 calculates the deviation between the sum of the line item percentiles and the overall contingency.
- Once the point at which the overall contingency is to be set has been entered into the cell F1, the value in cell F2 can be varied until the deviation is as close as possible to zero. It is usually impractical to achieve a perfect match but a deviation of a fraction of one percent is unlikely to be significant.
- Column G shows the allocated contingency as a percentage of the base estimate for each line.

This contingency allocation process only works after a simulation has been run. It needs a set of simulation data to be present in the model.

The percentile point used to choose the contingency for the separate items will always be lower than that used for the contingency on the total cost, so long as the point at which the contingency is set is above the mean value for the cost. This is because the total budget is at less risk than the individual line items due to the scope for variations in one item to balance out variations in another.

The base estimate and allocated work package values for this example are shown in **Table G1** with the overall contingency set at the 90th percentile and the allocation percentile adjusted to 79.2%, which brings the allocations within 0.1% of the overall contingency.

**Table G 1: Contingency allocation example**

Item	Base (\$)	Allocation (\$)	Allocation %
Provisions	2,929,452	456,251	16%
Preliminaries time independent	884,673	87,792	10%
Preliminaries time related	8,334,302	1,994,449	24%
Area 1	12,385,281	2,152,358	17%
Paths	4,441,160	648,706	15%
Viaduct	14,669,970	1,927,052	13%
Area 2	202,757	47,054	23%
Bridge	5,707,704	968,447	17%
Superstructure	493,602	46,205	9%

There are dangers in actually releasing contingency funds to work packages or parts of a project. If the allocations come to be seen as belonging to the areas to which they have been allocated, there is a real prospect that the balancing effect upon which the overall contingency calculation depends will be broken. However, the exercise is a valuable means of validating and checking the realism of the model.

An examination of the figures in **Table G1** shows which areas of the work are relatively more in need of contingency funds than others. The two where the largest proportional allocation falls are time dependent preliminaries and Area 2. Area 2 is a relatively small parcel of work so the amount of money involved is small even though the percentage is high, whereas the time dependent preliminaries is a large part of the project. At the other end of the scale, the superstructure line has been allocated the smallest amount relative to its base estimate value.

In the context of a real project, these observations will have meaning to the personnel involved. If they are consistent with what the team believes then they will confirm the validity of the model. If they are at odds with what the team believe then either there is a flaw in the model or the team's expectations are unrealistic. In either case, the discrepancy should be examined and resolved.

The same approach can be used to allocate contingency funds between two parties, say a proponent and a contractor. Instead of basing it on the lines of the summary estimate, exactly the same calculation of base estimate, simulated value, variation, and percentiles would be applied to the subtotals for the two parties.

## Appendix H – Common modelling errors

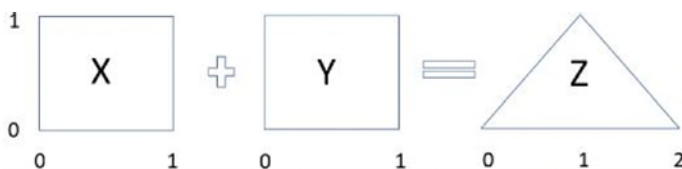
In Monte Carlo application, the step from inputting data into a model to obtaining realistic results is far from trivial. Unlike analytical calculations where gross errors often produce results which are obviously absurd, subtle bugs in Monte Carlo ‘reasoning’ easily give rise to answers which are completely wrong but still appear sufficiently reasonable to go unnoticed. Since few decision makers are familiar with the method, they will often regard anything with the right overall appearance, distributions and percentile values, as being valid.

This appendix explains some of the more common errors in the application of Monte Carlo simulation to estimate contingency, and suggests techniques to avoid the errors.

### Manipulating probability distributions as fixed numbers

A common mistake in a Monte Carlo simulation model is in calculating the sums or products of random variables.

Arithmetic operators cannot be applied to probability distributions as universally as with integers even if the probability distributions are identical. For example, the sum of two uniform independent (uncorrelated) Uniform (0,1) distributions might be expected to be a Uniform (0,2) distribution. But it is in fact a Triangle (0,1,2) distribution<sup>66</sup>:



Further, seeing that  $U(0,1) + U(0,1) = T(0,1,2)$  it would seem logical that  $T(0,1,2) - U(0,1)$  would take us back to our original  $U(0,1)$ . However, that is not the case; the answer is a symmetric distribution that looks somewhat normal, stretching from -1 to 2 with a peak at 0.5.

There is a common belief that the sum of the most likely costs of the components of an estimate

will equal the most likely value of the total cost. This is one of the most widespread misconceptions that interferes with understanding risk models.

When a set of quantities with uncertain values, represented by continuous distributions, are added, as with the components of an estimate, under most conditions the mean value of the total will be the sum of the mean values of the individual components. The same is not true for the most likely values, also called the modes. Nor is it true for percentile values such as the P10, P50 or P90.

In particular, adding up all the optimistic and pessimistic values of individual costs does not produce meaningful optimistic and pessimistic values for the total cost. Such calculations provide virtually no useful information at all. The chance of an extreme outcome, minimum or maximum, for a single part of an estimate or risk factor is very small. The chance of several uncorrelated items all taking their most extreme possible values is the product of the probability of each one taking on its most extreme value.

Calculations based on single values selected from a set of continuous distributions do not really make sense. It is usually necessary to work with intervals within a distribution's range to make such calculations. However, a heuristic illustration can be used to explain what happens when all the minima and maxima of a set of distributions are added together.

If the extreme values of a distribution are considered to have say a 1% chance of arising, a probability of 10-1, just for the purposes of this illustration, the probability that ten uncorrelated uncertain values will all fall at

<sup>66</sup> Vose D (2008) Risk Analysis: A Quantitative Guide

the extreme ends of their ranges in unison is  $(10^{-1})^{10}$  or  $10^{-10}$ . That is a microscopic likelihood, a probability of 0.0000000001.

Another way of looking at this is from a practical perspective. It is highly unlikely that 10 independent things will all go wrong at once, otherwise projects would be impossible to estimate and plan<sup>67</sup>. Indeed, the concept of contingency is based on the assumption that estimates for some components will be too high, some will be too low, but in aggregate, many of the highs and lows will net off such that overruns in some areas are paid for with savings from others.

So long as the ranges assessed for the inputs to a model are realistic and are not all correlated to one another, the chance of all of them falling at their most pessimistic values or their most optimistic values at the same time as one another is as good as zero. Calculations, proposals and business cases based on such cases are meaningless.

### **Adding inherent and contingent risk outputs together**

Consistent with the previous discussion, if a risk model is built using a combination of a line-item ranging approach to model inherent risks, with appropriate consideration of dependencies between items, and a risk event approach to model contingent risks, these cannot be added together to find total project percentile values, but must be modelled together to understand their aggregate effect.

It is often the case that a client will want to understand the potential impact of individual risks, or will want to split out inherent and contingent risks in order to determine how much may potentially be transferred to the contractor. This is perfectly acceptable however, all risks must first be modelled together in a single simulation. **Appendix G** outlines an appropriate technique to then allocate contingency between work packages or stakeholders on an equitable basis if this is necessary.

### **Not Truncating Distributions**

As discussed at **Appendix D**, unbounded and partially unbounded distributions may need to be constrained to remove the tail of the distribution so that nonsensical values are avoided. For example, costs can exclusively never go below zero (otherwise they are not a “cost”) so if a cost element is modelled using a distribution that can generate a negative value it must be truncated. It is also worthwhile validating whether the values chosen by the software simulation for any particular distribution actually conform to the analysts’ assessment of what they should be.

Typically an analyst determines the numerical values (optimistic, most likely, and pessimistic values) for any given model input and then chooses what is deemed to be the most appropriate probability distribution function (PDF). From these user-defined inputs the computer software will attempt to ensure that the PDF conforms to its “correct” shape within the bounds of the nominated numerical values while also ensuring that the total area under the PDF is one. This can give rise to unexpected values, particularly when ranges are very large. Occasional negative values for risk are probably not intended when the analyst is making an assessment of optimistic values.

**Table H1** demonstrates that the PDF that the software has ultimately created for the simulation may not be what the analyst intended. Sanity checking, or testing of distributions should be undertaken before running the model to determine if input parameter values need to be truncated or adjusted to ensure that they conform to what the analyst actually intended. This applies both to the optimistic and the pessimistic estimates.

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<sup>67</sup> Grey, S. (1995) Practical Risk Assessment for Project Management, John Wiley and Sons

**Table H 1: Theoretical minima and maxima for a selection of distributions**

Probability Distribution Function	User-defined values			Software-determined values	
	Optimistic	Most Likely	Pessimistic	Theoretical minimum	Theoretical maximum
Triangle	80	100	130	80	130
Trigen	80	100	130	61.3	151.6
Pert	80	100	130	80	130
PerItAlt	80	100	130	74	161.1
Triangle	50	100	200	50	200
Trigen	50	100	200	-2.6	267.1
Pert	50	100	200	50	200
PertAlt	50	100	200	41.6	336.9

**Including an excessive number of line items in a model**

As discussed in **Appendix F** probabilistic models should be limited to between 20 and 40 inputs in order to maintain the validity of the results.

The greater the number of independent (i.e. uncorrelated) inputs that are put into a model, the tighter the distribution will become as it converges to a normal distribution and the extreme tails of heavily-skewed marginal distributions are lost.

It should also be noted that it may not always be appropriate to simply aggregate, or roll-up, the cost model into a standard breakdown because certain types of cost may warrant more detail than others. As discussed at Section 6, the estimate should be summarised in a form that separates groups of costs that are subject to separate quantity and rate uncertainties. This may mean aggregating costs by geographical area, greenfield and brownfield, types of work, subcontracts, or some other logical structure that reflects the separate uncertainties or type of risk exposure.

**Including low probability, high consequence risks**

As discussed at [Section 4.4](#), using a weighted impact (i.e. probability x impact) to calculate the contingency required for events that could have very large undesirable consequences, is rarely satisfactory. Weighting risks by their likelihood of occurring only works when there are many small or medium events to be covered and they are independent of one another. Funding for very high impact risks with an appreciable probability of occurring is not suited to the way contingency funds are determined for a project that is subject to the net effect of a large number of independent small and medium scale risks.

If there is a need to embark on work that could be subject to a very large cost increase due to an identified event, this is best managed as a stand-alone contingent funding requirement with an agreed trigger and controls on the release of the funds. To incorporate it into a general project contingency only serves to hide the nature of the requirement and obscure the special character of the costs involved.

Extremely rare force majeure-type events (extreme weather, terrorist attacks, etc) are all conceivable but very unlikely. These are generally regarded as normal risks associated with what is sometimes called 'business as usual'. There may be insurance held against some of these but some will simply be accepted due to their rarity.

# Appendix I – Alternate approaches to eliciting expert opinion

Professor Mark Burgman became the foundation director of the Australian Centre of Excellence for Risk Analysis (ACERA - now Centre for Excellence for Biosecurity Risk Analysis) on its establishment in 2006.

While noting that estimates can be wrong in many ways, Burgman presents the following list as a start point to break down the quality of an expert's judgment.

- Accuracy measures how close an expert's quantitative estimate is to the truth. Accuracy can be measured by the difference between an expert's estimate and the correct answer. Over several questions, it may be the average difference.
- Bias measures the tendency of an expert to deviate consistently from the truth in a single direction, either too high or too low. Bias can only be measured over the answers to several questions.
- Calibration is the frequency with which uncertainty intervals enclose the truth, compared to the frequency with which the expert expects them to. Calibration can be measured by counting, over several questions, the frequency with which the expert's intervals enclose the truth.
- Reliability is a property of an expert. It is the degree to which an expert's estimates are repeatable and stable.

Leading from the dot points above, if an expert is routinely close to the truth but provides very wide margins of confidence (i.e. estimates are poorly calibrated), the judgements are accurate but uninformative. If an expert confidently provides narrow bounds but is routinely far from the truth, the judgements are simply misleading. Good estimates are accurate and well calibrated and a good judge is accurate, unbiased, well calibrated and reliable.

Professor Burgman has published widely on the topic of eliciting expert opinion and recommends a structured procedure termed a four-point format. The approach attempts to both elicit an interval to capture uncertainty, and to mitigate the overconfidence typically observed in expert estimates. Readers of this guidance note who wish to obtain a deeper understanding of dealing with experts may wish to investigate some of his relevant work<sup>68, 69</sup>.

In the four-point approach, the bounds are intentionally asked for before the best estimate, to make people think about extreme values, and to prevent them first thinking about and then anchoring on a best estimate. The recommended question format is:

- First, consider all the things that might lead you to conclude a low number. With these things in mind, realistically, what do you think is the smallest plausible value?.
- Second, consider all the things that might lead you to conclude a high number. With these things in mind, realistically, what do you think is the largest plausible value?.
- Third, thinking of all the things that contribute to your estimate, realistically, what is your best estimate?
- Finally, how confident are you that the interval your created, from lowest to highest, captures the true value? (This should be expressed as a number between 50 per cent and 100 per cent.).

Once responses have been gathered, the final step is rescale the interval (provided at the last question) so that it is equivalent to a reasonably high level of confidence. For example, if an interval of 60 per cent was provided, it is rescaled to be wider, typically so that it encompasses 80 percent of an expert's beliefs. This is

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<sup>68</sup> Speirs-Bridge, A., Fidler, F., McBride, M., Flander, L., Cumming, G., and Burgman, M. (2010). Reducing Overconfidence in the Interval Judgments of Experts. *Risk Analysis* 30, 512-523. doi: 10.1111/j.1539-6924.2009.01337.x

<sup>69</sup> Burgman M (2015) *Trusting Judgements, How to Get the Best out of Experts*, Cambridge University Press

done in order to compare and combine intervals from individual experts. It is also done because intervals with low confidence (say 50 per cent) are usually not very informative.

Estimates are scaled, or normalised, using linear extrapolation to absolute lower ( $\alpha_{abs}$ ) and upper ( $\beta_{abs}$ ) bounds such that:

$$\alpha_{abs} = \gamma - (\gamma - \alpha)(c/p)$$

$$\beta_{abs} = \gamma + (\beta - \gamma)(c/p)$$

where ( $\gamma$ ) is the most likely value, ( $c$ ) is the required possibility level, and ( $p$ ) is the expert's stated confidence.

For example, if an expert provided interval values of 90 (smallest plausible), 100 (best assessments), and 115 (highest plausible) with a 60 per cent confidence level and we wanted to rescale the intervals so that they represent the equivalent of an 80 per cent confidence level, the absolute lower and upper bounds become:

$$\alpha_{abs} = \gamma - (\gamma - \alpha)(c/p) = 100 - (100-90)(80/60) = 86.67$$

$$\beta_{abs} = \gamma + (\beta - \gamma)(c/p) = 100 + (115-100)(80/60) = 120$$

The ACERA Elicitation Tool (comprising Process Manual<sup>70</sup> and User Manual<sup>71</sup> 2010) provide comprehensive guidance on the conduct of risk workshops. While this manual was developed within the context of ecological risks, the Elicitation Tool can be used for expert panel risk assessment in any domain where expert opinion can be articulated in the form of interval estimates. The scaling method is approximate and the range derived from it should be considered carefully before it is used in further analysis.

The department considers that either the approach described earlier at [Section 5.4](#) of this guidance note, or the ACERA Elicitation Tool are suitable methods for eliciting expert opinion as part of a broader risk workshop process.

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<sup>70</sup> Australian Centre of Excellence for Risk Analysis (2010) Elicitation Tool – Process Manual available at [http://cebra.unimelb.edu.au/publications/acera\\_reports/acera\\_1](http://cebra.unimelb.edu.au/publications/acera_reports/acera_1)

<sup>71</sup> Australian Centre of Excellence for Risk Analysis (2010) Elicitation Tool – User Manual available at [http://cebra.unimelb.edu.au/publications/acera\\_reports/acera\\_1](http://cebra.unimelb.edu.au/publications/acera_reports/acera_1)



## Appendix J – Definitions and abbreviations

Term	Definition
<b>Agency</b>	A state or territory government body that is generally responsible for delivering land transport infrastructure project.
<b>Assumption</b>	A documented, cost-related factor that, for the purpose of developing a base cost estimate is considered to be true, real or certain.
<b>Base Date</b>	The reference date from which changes in conditions, (including rates and standards) can be assessed. In the context of a base estimate, it is the date for which the rates included in the cost estimate reflect current market conditions.
<b>Base Estimate</b>	The sum of the construction costs and client's costs at the applicable base date. It represents the best prediction of the quantities and current rates which are likely to be associated with the delivery of a given scope of work. It should not include any allowance for risk (contingency) or escalation.
<b>BCR</b>	The Benefit Cost Ratio (BCR) is the ratio that represents the benefits over costs and is represented as a single number. Further guidance on BCR can be found on the <a href="#">Australian Transport Assessment and Planning (ATAP)</a> website.
<b>Client Costs</b>	In this guidance note, 'client' is the project proponent. Client costs are the costs incurred by the proponent (e.g. public sector delivery agency) to develop and deliver a project.
<b>Construction Costs</b>	The costs required to complete the activities or tasks associated with the construction elements of a project.
<b>Contingency</b>	<p>An amount added to an estimate to allow for items, conditions, or events for which the state, occurrence, or effect is uncertain and that experience shows will likely result, in aggregate, in additional costs<sup>72</sup>. This does not include escalation.</p> <p>As per Appendix B of NoA: <i>"The component of a Project's cost in excess of the Project Base Estimate that accounts for, or reflects, risk"</i>.</p> <p>For further information on contingency refer to <b>Guidance Notes 3A and 3B</b>.</p>
<b>Contractor Direct Costs</b>	All contractor's costs directly attributable to a project element including, but not limited to, plant, equipment, materials, and labour.
<b>Contractor Indirect Costs</b>	Costs incurred by a contractor to perform work that are not directly attributable to a project element. These generally include costs such as preliminaries, supervision, and general and administrative costs.
<b>Escalation</b>	The component of a project's total cost at any point in time that reflects changes in prices and costs since the base cost estimate date. Escalation is added to the project cost to obtain the outturn cost. Escalation aspects do not form part of the scope of this document. For further information refer to <b>Guidance Note 4 - Escalation</b> .

<sup>72</sup> AACE International, Recommended Practice 10S-90, Cost Engineering Terminology, accessed 19 October 2022  
<<https://web.aacei.org/docs/default-source/rps/10s-90.pdf>>

Term	Definition
<b>Escalation Rate</b>	The department derives escalation rates from actual or forecast composite index series that reflect the characteristics of infrastructure projects, where the escalation rate in any financial year is calculated from the average of the composite quarterly indexes for that financial year divided by the average of the composite quarterly indexes for the previous financial year.
<b>Estimator</b>	The person or organisation that prepares a cost estimate.
<b>First Principles Estimate</b>	The method of preparing a cost estimate by breaking down the project into a work breakdown structure and determining rates and quantities for each component. The cost estimate is the summation of each component.
<b>Jurisdiction</b>	An Australian state or territory.
<b>Labour</b>	Effort expended by people for wages or salary.
<b>Margin</b>	An allowance that includes the construction contractor's corporate overheads and profit.
<b>Material</b>	An article, material, or supply brought to a construction site by the contractor or a subcontractor for incorporation into the work. Also includes any items brought to the site preassembled from articles, materials or supplies.
<b>NoA</b>	The Notes on Administration for Land Transport Infrastructure Projects 2019-2024 (NoA), provide administrative detail to support the National Partnership Agreement (NPA) and apply to all Projects funded, or proposed to be funded under Part 3 (Investment Projects) and Part 7 (Black Spot Projects) of the National Land Transport Act 2014 (NLT Act).
<b>NPA</b>	<p>National Partnership Agreement on Land Transport Infrastructure Projects (NPA). The NPA supports the delivery of infrastructure projects and sets out how the Australian Government and states will work together to deliver infrastructure projects for the benefit and wellbeing of Australians.</p> <p>The NPA covers projects administered under the National Land Transport Act 2014 (NLT Act) each state has a separately agreed schedule to the NPA which indicate the levels of funding the Australian Government intends to provide for land transport infrastructure investments. These schedules are updated following the Federal Budget each year, and as required.</p>
<b>Outturn Cost</b>	Outturn cost is the summation of the base cost, contingency and the total escalation (it is the nominal total project cost). The department's Project Cost Breakdown (PCB) template can be used to calculate escalation and outturn costs. In economic terms, non-escalated costs are often referred to as real costs while outturn costs are often referred to as nominal costs.
<b>Overhead(s)</b>	A cost or expense inherent in the performing of an operation, (e.g. engineering, construction, operating, or manufacturing) which cannot be charged or identified with a part of the work, product or asset and, therefore, must be allocated on some arbitrary basis believed to be equitable, or handled as a business expense independent of the volume of production. These costs are considered when determining the cost of business. (e.g. machine maintenance, company accounting costs etc.)

Term	Definition
<b>Project Cost Breakdown (PCB) Template</b>	The PCB template is provided by the department and is updated annually to reflect the latest escalation rates for road and rail projects.
<b>Project Cost</b>	The base estimate cost plus an allowance for contingency and generally prefixed by P50 or P90 to represent the level of contingency included. The project cost reflects costs as of the base estimate date. This does not include escalation.
<b>Plant</b>	All machines, motor vehicles, appliances and things (for example, scaffolding and formwork) used or in use in the execution of the work, but not materials, plant, equipment intended to form part of the final work.
<b>Project Proposal Report (PPR)</b>	A statement detailing the scope and benefits of the project submitted by proponents as part of the project approval process for funding under the Infrastructure Investment Program (IIP).
<b>Project Scope</b>	The work that must be performed to deliver a product, service or result with the specified features and functions.
<b>Subcontractor</b>	A contractor that enters into a subcontract and assumes some of the obligations of the primary contractor.
<b>Sunk Costs</b>	Costs which have already been incurred, such as investigation, research, and design costs. Sunk costs are included in an outturn cost.
<b>Work Breakdown Structure (WBS)</b>	A way of organising a project using a hierarchical breakdown of the activities required to complete the project. The WBS organises and defines the total scope of the project.

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